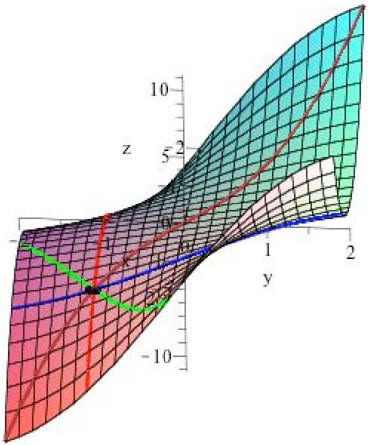
Intuition about Derivatives



Suppose an object is at the illustrated point $P = \big(x_0, y_0, f(x_0, y_0)\big)$.

- Object may move in any direction.
- Direction of motion determines how function changes.
- Infinitely many rates of change are associated with P.
- Infinitely many lines tangent to $f(x, y)$ at P, each heading off in different directions.

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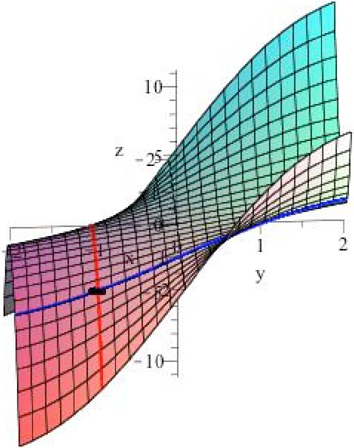

For now:

Restrict attention to the x and y directions

- that is, to moving in directions parallel to the x or the y axes.

Recall from Single-Variable Calculus

- Recall that in single-variable calculus, the derivative of $f(x)$ at $x = a$ is
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and the function which gives the derivative at any x is
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- What does the derivative measure?
  - $f'(a)$ is the slope of the line tangent to $f(x)$ at $\big(a, f(a)\big)$.
  - $f'(a)$ is the rate the function is changing at $x = a$ as you move along the curve in the positive x direction.