Daily WW #2

Choose which to compare to

$$\int_1^\infty \frac{dx}{x} \qquad \int_1^\infty \frac{dx}{x^2} \qquad \int_1^\infty \frac{dx}{x^3} \qquad \int_1^\infty \frac{dx}{\sqrt{x}}$$

using the Comparison Test. Determine convergence vs divergence.

1. $0 \le \displaystyle\int_1^\infty \frac{dx}{x^2+5} \le \int_1^\infty \frac{dx}{x^2} \quad \left(x^2+5 > x^2 \Rightarrow 0 < \frac{1}{x^2+5} < \frac{1}{x^2}\right)$
   $\displaystyle\int_1^\infty \frac{dx}{x^2}$ converges $\Rightarrow \displaystyle\int_1^\infty \frac{dx}{x^2+5}$ must converge.

2. $\displaystyle\int_1^\infty \frac{2+e^{-x}}{x}\, dx \ge \int_1^\infty \frac{dx}{x} \quad \left(2+e^{-x} > 2 \Rightarrow \frac{2+e^{-x}}{x} > \frac{1}{x}\right)$
   $\displaystyle\int_1^\infty \frac{dx}{x}$ diverges $\Rightarrow \displaystyle\int_1^\infty \frac{2+e^{-x}}{x}\, dx$ must diverge.

Daily WW #2, continued

Choose which to compare to

$$\int_1^\infty \frac{dx}{x} \qquad \int_1^\infty \frac{dx}{x^2} \qquad \int_1^\infty \frac{dx}{x^3} \qquad \int_1^\infty \frac{dx}{\sqrt{x}}$$

using the Comparison Test. Determine convergence vs divergence.

1. $0 \leq \int_1^\infty \frac{x}{x^3+1}\,dx \leq \int_1^\infty \frac{dx}{x^2}$ $\left(x^3+1 > x^3 \Rightarrow 0 < \frac{x}{x^3+1} < \frac{x}{x^3}\right)$
   $\int_1^\infty \frac{dx}{x^2}$ converges $\Rightarrow \int_1^\infty \frac{x}{x^3+1}\,dx$ must converge.
2. $\int_1^\infty \frac{2+\cos(x)}{\sqrt{x}}\,dx \geq \int_1^\infty \frac{dx}{\sqrt{x}}$
   $\left(2+\cos(x) > 2-1 \Rightarrow \frac{2+\cos(x)}{\sqrt{x}} > \frac{1}{\sqrt{x}}\right)$
   $\int_1^\infty \frac{dx}{\sqrt{x}}$ diverges $\Rightarrow \int_1^\infty \frac{2+\cos(x)}{x}\,dx$ must diverge.

Goal:

- We've seen that in some cases, adding up an infinite number of numbers – an infinite sum– makes sense:

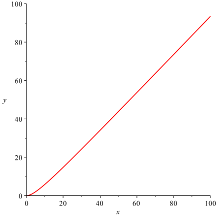
$$\sum_{k=1}^{\infty} \left(\frac{1}{2}\right)^k \overset{\text{seems to}}{=} \quad 1$$

$$0 < \sum_{n=2}^{\infty} \frac{1}{n^2} \leq \quad 1$$

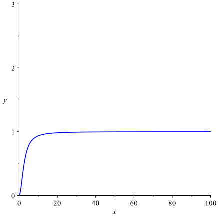
- **Goal:** Work toward formalizing the idea of an **infinite sum**.
- In order to do this, first must discuss infinite lists of numbers – **Sequences**.

Indeterminate Form $\frac{\infty}{\infty}$

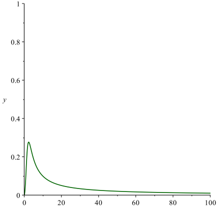
$$\lim_{x\to\infty} \frac{x^2}{x+7} = \frac{\infty}{\infty} = \infty$$



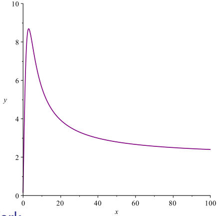
$$\lim_{x\to\infty} \frac{x^2}{x^2+7} = \frac{\infty}{\infty} = 1$$



$$\lim_{x\to\infty} \frac{x^2}{x^3+7} = \frac{\infty}{\infty} = 0$$



$$\lim_{x\to\infty} \frac{2x^2+40x+1}{x^2+7} = \frac{\infty}{\infty} = 2$$



Sequences:

- **Formal Definition:** A **sequence of real numbers** $\{a_k\}$ is a function whose domain is the set of integers, starting with some integer n_0 (often 0 or 1). The **terms** of the sequence are the individual outputs of the function: for each positive integer $k \geq n_0$, the output a_k is called the ***k*th term** of the sequence.

  Example The function $a(k) = 2^k$, for $k = 0, 1, 2, 3, \ldots$ defines the sequence
  $$\{a_k\}_{k=0}^{\infty} = \{1, 2, 4, 8, 16, \ldots\}$$

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  $$\{a_k\}_{k=0}^{\infty} = \{1, 2, 4, 8, 16, \ldots\}$$

  In this example, since $1 = a(0)$, 1 is referred to as the **0th term** and denoted a_0. Since $2 = a(1)$, 2 is the **first term**, a_1, etc. We call 2^k the **general term** a_k.

Question:

Find a symbolic expression for the general term a_k of the sequence

$$\{0, 3, 6, 9, 12, 15, \ldots\}$$

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Answer: $\{0, 3, 6, 9, 12, 15, \ldots\} = \{3k\}_{k=0}^{\infty}$

Question We Can Ask About Sequences:

What are the terms of the sequence doing in the long run? Are they increasing without bound? Bouncing around? Approaching some specific value?

- In other words,

  Do the terms of the sequence **converge** to a limiting value L, or do they diverge, either by approaching $\pm\infty$ or by not approaching anything at all?

- To determine convergence/divergence, use the same basic idea behind convergence and divergence as we always have – **but** . . .
- . . . be aware difference is that our domain is just integers rather than all real numbers.