# Daily WeBWorK, Problem 2

(a) $6x^2 + 6y^2 - z = 0$

- $x$-intercept$=$$y$-intercept$=$$z$-intercept: $(0, 0, 0)$
- $xy$-trace: $6x^2 + 6y^2 = 0 \Rightarrow x = y = 0 \Rightarrow (0, 0, 0)$
- $xz$-trace: $z = 6x^2 \Rightarrow$ parabola opening along $z$-axis
- $yz$-trace: $z = 6y^2 \Rightarrow$ parabola opening along $z$-axis
  - 2 parabolic traces $\Rightarrow$ **paraboloid**
- We now have a sense of something opening up around the $z$-axis, but in what shape?
  - What are the cross-sections parallel to the $xy$-plane?
  - Section in $z = 1$: $6x^2 + 6y^2 = 1 \Rightarrow$ circle
  - Section in $z = 2$: $6x^2 + 6y^2 = 2 \Rightarrow$ circle
  - In any plane $z = k$ with $k > 0$, circle
  - No sections below the $xy$-plane

  Circular sections in third direction $\Rightarrow$ **circular paraboloid**

  Since it opens around the $z$-axis, that is its axis of symmetry

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(b) $-9x^2 + 6y^2 - z^2 = 0$

- $x$-intercept=$y$-intercept=$z$-intercept: $(0, 0, 0)$
- $xy$-trace: $-9x^2 + 6y^2 = 0 \Rightarrow 9x^2 = 6y^2 \Rightarrow y = \pm\sqrt{\frac{9}{6}}x \Rightarrow$ 2 lines
- $xz$-trace: $z^2 = -9x^2 \Rightarrow$ NO trace in the $xz$-plane
- $yz$-trace: $z^2 = 6y^2 \Rightarrow z = \pm\sqrt{6}y \Rightarrow$ 2 lines
  - 2 pairs of crossing lines $\Rightarrow$ **maybe a cone?**
- Since no trace in $xz$-plane, no idea what's going on parallel to the "side wall".
  - What are the cross-sections parallel to the $xz$-plane?
  - Section in $y = \pm 1$: $-9x^2 + 6 - z^2 = 0 \Rightarrow 9x^2 + z^2 = 6 \Rightarrow$ ellipses
  - Section in $y = \pm 2$: $-9x^2 + 24 - z^2 = 0 \Rightarrow 9x^2 + z^2 = 24 \Rightarrow$ ellipses
  - In any plane $y = \pm k$: ellipses

Elliptical sections in third direction $\Rightarrow$ **elliptical cone**

Since it has elliptic cross-sections parallel to the $xz$plane, it is symmetric around the $y$-axis

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(c) $6y^2 - 6x^2 - 5z^2 + 1 = 0 \Rightarrow 6x^2 - 6y^2 + 5z^2 = 1$

- $x$-intercepts: $(\pm\sqrt{\frac{1}{6}}, 0, 0)$, $y$-intercept: none, $z$-intercept: $(0, 0, \pm\sqrt{\frac{1}{5}})$
- $xy$-trace: $6x^2 - 6y^2 = 1 \Rightarrow$ hyperbola opening along $x$, not touching $y$
- $xz$-trace: $6x^2 + 5z^2 = 1 \Rightarrow$ ellipse
- $yz$-trace: $5z^2 - 6y^2 = 1 \Rightarrow$ hyperbola opening along $z$, not touching $y$
  - 2 hyperbolas $\Rightarrow$ **hyperboloid**
- Since it has a trace in the $xz$-plane, it's a hyperboloid of 1 sheet. It is symmetric around the axis it doesn't touch – the $y$-axis.

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(d) $\Rightarrow -5x^2 - 9y^2 + z^2 = 1$

- $x$-intercepts: none, $y$-intercept: none, $z$-intercepts: $(0, 0, \pm 1)$
- $xy$-trace: $-5x^2 - 9y^2 = 1 \Rightarrow$ none
- $xz$-trace: $z^2 - 5x^2 = 1 \Rightarrow$ hyperbola opening along $z$, not touching $x$
- $yz$-trace: $z^2 - 9y^2 = 1 \Rightarrow$ hyperbola opening along $z$, not touching $y$
  - 2 hyperbolas $\Rightarrow$ **hyperboloid**
- Since it **does not** have a trace in the $xy$-plane, it's a hyperboloid of 2 sheet.

  It is symmetric around the axis it **does** touch – the $z$-axis.

# Daily WeBWorK, Problem 2

(e) $\Rightarrow 9y^2 + 9z^2 = 1$

- $x$-intercepts: none, $y$-intercepts: $(0, \pm\frac{1}{3}, 0)$, $z$-intercepts: $(0, 0, \pm\frac{1}{3})$
- $xy$-trace: $9y^2 = 1 \Rightarrow (0, \pm\frac{1}{3}, 0)$
- $xz$-trace: $9z^2 = 1 \Rightarrow (0, 0, \pm\frac{1}{3})$
- $yz$-trace: $9y^2 + 9z^2 = 1 \Rightarrow$ circle

- Because $y$ and $z$ are connected but $x$ is free, this is a cylinder extending infinitely in the $x$ direction (and hence $x$ is the axis of symmetry).
- Because all cross-sections parallel to the $yz$-plane are circles, it is a circular cylinder.

# Functions of Two Variables

**Example:** Below are the graphs of four surfaces. Two of them are the graphs of $f_1(x, y) = [(x^2 + y^2) - 2]^3$ and $f_2(x, y) = y^2 \sin(x)$. Which two?