WeBWorK #2

Write out the sigma notation for the right sum with $n = 4$ that approximates the signed area between $f(x) = \sqrt{x+1}$ and the x-axis on $[2, 3]$. Do this in such a way that the only variable is k. (Go from $k = 1$ to $k = 4$).

That is, Find R_4 for $\int_2^3 \sqrt{x+1}\, dx$.

$a = 2,\ b = 3 \Rightarrow \Delta x = \frac{b-a}{n} = \frac{3-2}{4} = \frac{1}{4}$

Thus the partition of $[2, 3]$ is $\left\{2, \underbrace{2+\frac{1}{4}, 2+\frac{1}{2}, 2+\frac{3}{4}, 3}_{\text{Right endpoints}}\right\}$

Conclusions so far:

- The width of each rectangle $= \Delta x = \frac{1}{4}$
- The heights of each rectangle $= f(\text{right endpoints})$

$$R_4 = \frac{1}{4}\left[f\left(2+\frac{1}{4}\right) + f\left(2+\frac{1}{2}\right) + f\left(2+\frac{3}{4}\right) + f\left(3\right)\right]$$

WeBWorK #2

Find R_4 for $\int_2^3 \sqrt{x+1}\,dx$.

$$R_4 = \frac{1}{4}\left[\underbrace{f\left(2+\frac{1}{4}\right)}_{k=1} + \underbrace{f\left(2+\frac{1}{2}\right)}_{k=2} + \underbrace{f\left(2+\frac{3}{4}\right)}_{k=3} + \underbrace{f\left(3\right)}_{k=4}\right]$$

Can see that for each k from $k = 1$ to $k = 4$, the inside of the function is $2 + \frac{k}{4}$. Thus

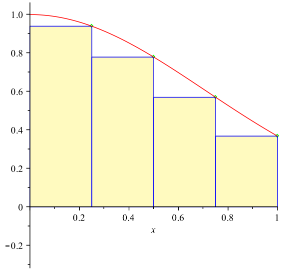
$$R_4 = \sum_{k=1}^{4} \frac{1}{4} f\left(2 + \frac{k}{4}\right)$$

But we need to say what f *does* to each input!

$$R_4 = \sum_{k=1}^{4} \frac{1}{4}\sqrt{(2+\frac{k}{4})+1} = \sum_{k=1}^{4} \frac{1}{4}\sqrt{3+\frac{k}{4}}$$

Question about approximations:

Last time, looked at $\mathcal{I} = \int_0^{1/2} e^{-x^2}\, dx$. Found:



Because f is decreasing, R_4 is an underestimate

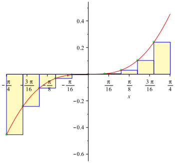
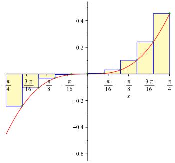
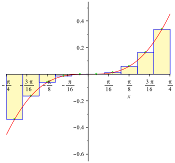
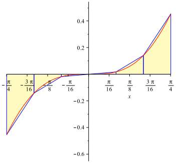
Also found

$$\begin{aligned}\mathcal{I} &\approx R_4\\ &\approx \frac{1}{8}\cdot e^{-(1/8)^2} + \frac{1}{8}\cdot e^{-(1/4)^2}\\ &\quad + \frac{1}{8}\cdot e^{-(3/8)^2} + \frac{1}{8}\cdot e^{-(1/2)^2}\\ &\approx 0.4464406673\end{aligned}$$

Question: Can we find the error in using R_4 to approximate $\mathcal{I}$ exactly?

Approximating Integrals with Sums

Example: If a function f is always increasing on $[a, b]$ *(Monotonic)*

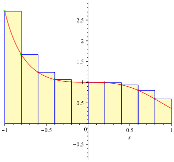
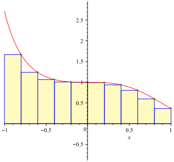
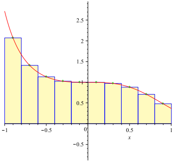
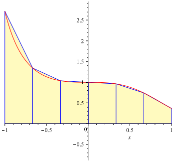
Left Sum	Right Sum	Midpoint Sum	Trapezoidal
			
Underestimates	Overestimates	Neither (?)	Neither

If f is increasing on $[a, b]$, L_n will underestimate and R_n will over-estimate.

$$L_n \leq \mathcal{I} \leq R_n.$$

Approximating Integrals with Sums

Example: If f is always decreasing on $[a, b]$ *(Monotonic)*

Left Sum	Right Sum	Midpoint Sum	Trapezoidal
			
Overestimates	Underestimates	Neither(?)	Neither

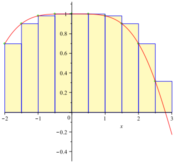
If f is decreasing, L_n will overestimate and R_n will under-estimate.

$$R_n \leq \mathcal{I} \leq L_n.$$

Approximating Integrals with Sums

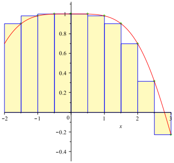
Example: If f is always concave down on $[a, b]$:

Left Sum



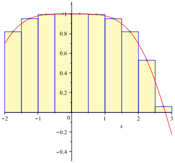
Neither

Right Sum



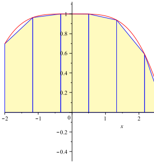
Neither

Midpoint Sum



Overestimates (?)

Trapezoidal



Underestimat

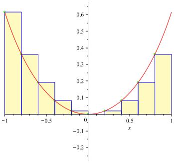
Whenever f is concave down on $[a, b]$, M_n will overestimate and T_n will under-estimate.

$$T_n \le \mathcal{I} \le M_n.$$

Approximating Integrals with Sums

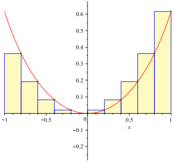
Example: If f is always concave up on $[a, b]$:

Left Sum



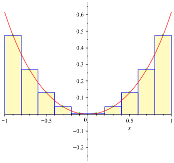
Neither

Right Sum



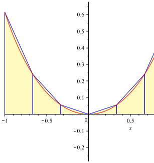
Neither

Midpoint Sum



Underestimates (?)

Trapezoidal



Overestimate

Whenever f is concave up on $[a, b]$, M_n will underestimate and T_n will over-estimate.

$$M_n \leq \mathcal{I} \leq T_n.$$

Graph of $e^{\cos(x)}$