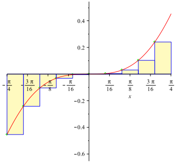
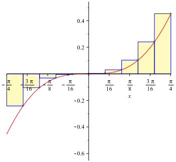
Recall from last time - If f is monotonic

If f is increasing on $[a,b]$,

$L_n \leq \mathcal{I}$ $\qquad$ $R_n \geq \mathcal{I}$



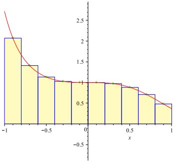


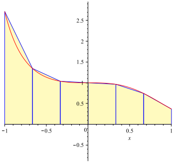
If f is increasing on $[a,b]$

$$L_n \leq \mathcal{I} \leq R_n.$$

If f is decreasing on $[a,b]$,

$L_n \geq \mathcal{I}$ $\qquad$ $R_n \leq \mathcal{I}$





If f is decreasing on $[a,b]$

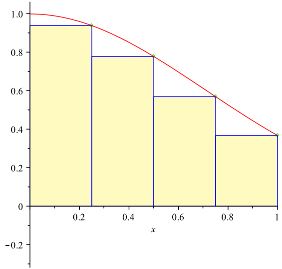
$$R_n \leq \mathcal{I} \leq L_n$$

Thus if f is **monotonic** (that is, always increasing *or* always decreasing), we can bound the error involved in using either L_n or R_n to approximate $\mathcal{I}$.

$$\begin{aligned} E_{L_n} = \left|\mathcal{I} - L_n\right| &\leq \left|R_n - L_n\right| \\ \underbrace{E_{R_n} = \left|\mathcal{I} - R_n\right|}_{\text{error}} &\leq \underbrace{\left|R_n - L_n\right|}_{\text{an error bound}} \end{aligned}$$

Recall example:

If $\mathcal{I} = \int_0^{1/2} e^{-x^2}\, dx$,



$$\begin{aligned} \mathcal{I} &\approx R_4 \\ &\approx \frac{1}{8}\cdot e^{-(1/8)^2} + \frac{1}{8}\cdot e^{-(1/4)^2} \\ &\quad + \frac{1}{8}\cdot e^{-(3/8)^2} + \frac{1}{8}\cdot e^{-(1/2)^2} \\ &\approx 0.4464406673 \end{aligned}$$

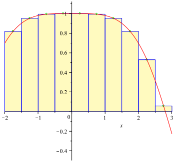
Question: We know that $E_{R_4} \leq \left| L_4 - R_4 \right|$.

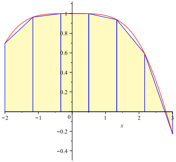
To find a bound on how well or poorly 0.44644 approximates $\mathcal{I}$, do we need to find L_4 as well?

Recall - If f is always conc up or conc down

If f is concave down on $[a, b]$,

$M_n \geq \mathcal{I}$ $\qquad$ $T_n \leq \mathcal{I}$



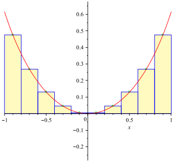


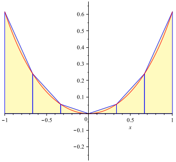
If f is conc down on $[a, b]$

$T_n \leq \mathcal{I} \leq M_n$

If f is concave up on $[a, b]$,

$M_n \leq \mathcal{I}$ $\qquad$ $T_n \geq \mathcal{I}$



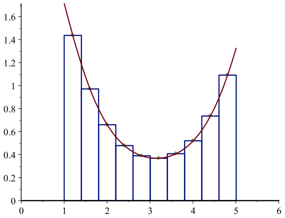


If f is conc up on $[a, b]$

$M_n \leq \mathcal{I} \leq T_n$

Example:

Let $\mathcal{I} = \displaystyle\int_1^5 e^{\cos(x)}\, dx$.



Notice that $e^{\cos(x)}$ is concave up on $[1, 5]$

It turns out that

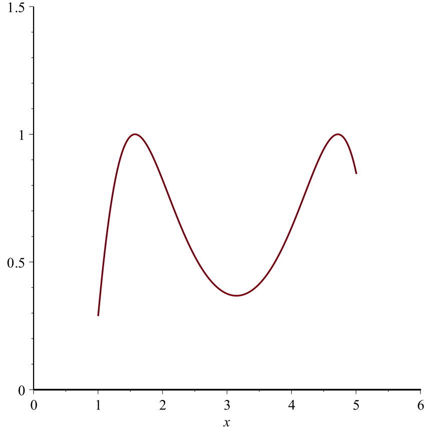
$$M_{10} \approx 2.823611872.$$

Thus

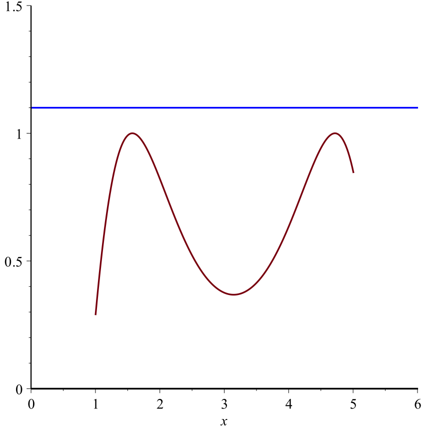
$$\int_0^{10} e^{\cos(x)}\, dx \approx 2.823611872.$$

But how close **is** $\mathcal{I}$ to 2.823611872?

To find M, look at graph of $f''(x)$ on $[1, 5]$:



To find M, look at graph of $f''(x)$ on $[1, 5]$:



Because $0 \le f''(x) \le 1.1$, can use $M = 1.1$