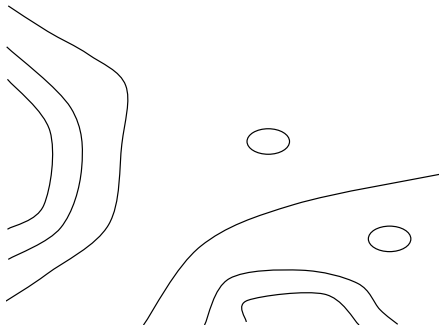
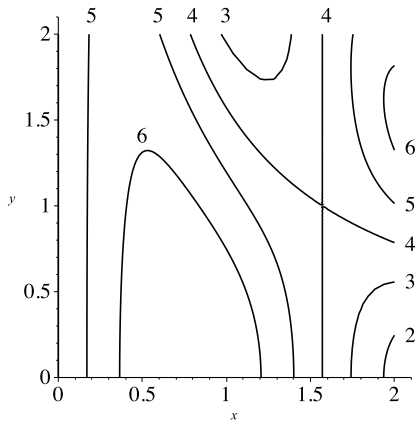
Group Problem Set 3

1. Suppose that the contour plot below represents the temperature in a room. If it is winter, identify likely positions for a heating vent and a window. Speculate on what the remaining features of the contour plot may represent.



2. Show that $\lim_{(x,y)\to(0,0)} \dfrac{12xy^3}{4x^2+2y^6}$ does not exist by finding paths that lead to different limits.

3. Use the given contour plot to estimate the equation of the plane tangent to $f(x,y)$ at the point $(1.4, 0)$. Then use that the tangent plane gives a linear approximation to estimate $f(1.3, 0.1)$.



4. Suppose that the number of units manufactured by a large firm is modeled by the production function

$$p(k,l) = 16k^{1/3}l^{2/3},$$

where k measures the firm's capital (in millions of dollars) and l measures the firm's labor force (measured in thousands of workers). Suppose that currently the firm has 2000 workers and the labor force is increasing at a rate of 40 workers per year, *and* that the firm has \$5000000 in capital, but the capital is decreasing at the rate of \$100000 per year. Determine the current rate of change in production, in units per year.

5. Let $f(x, y) = x^3 - 3xy + y^3$

   (a) Find and classify all critical points of $z = f(x, y)$

   (b) Consider the traces of the surface in the planes $y = kx$ for all k,

$$\begin{aligned} g_k(x) &= f(x, kx) \\ &= x^3 - 3kx^2 + (kx)^3 \end{aligned}$$

   Classify the behavior of $g_k(x)$ at $x = 0$ for all k. That is, are there some values of k for which $(0, 0)$ is a local maximum of g_k? A local minimum? Are there some values of k for which $(0, 0)$ is an inflection point? (Notice that this problem uses only techniques from Calculus 1).

   (c) Use your results from (b) to gain a deeper understanding of one of your results in (a), and explain through a clear and concise combination of description and sketches.