

For each integral,  $\int_a^b f(x) dx$ , explain why  $\int_a^b f(x); dx$  is improper, and determine whether the integral converges or diverges.

1)  $\int_1^\infty \frac{1}{x^3} dx$

2)  $\int_1^\infty \frac{1}{x} dx$

3)  $\int_0^1 \frac{1}{x^3} dx$

4)  $\int_1^\infty \frac{1}{x^p} dx$  where  $p > 1$

5)  $\int_0^1 \frac{1}{x^p} dx$  where  $p > 1$

Look back at Problems 1, 2, and 4.

1. Is it necessary that  $f(x)$  converge to 0 as  $x \rightarrow \infty$  in order for  $\int_a^b f(x) dx$  to converge to a finite number?

2. If  $f(x)$  converges to 0 as  $x \rightarrow \infty$ , will  $\int_a^b f(x) dx$  automatically converge to a finite number? That is, is  $f(x) \rightarrow 0$  a sufficient condition for  $\int_a^b f(x) dx$  to converge to a finite number?