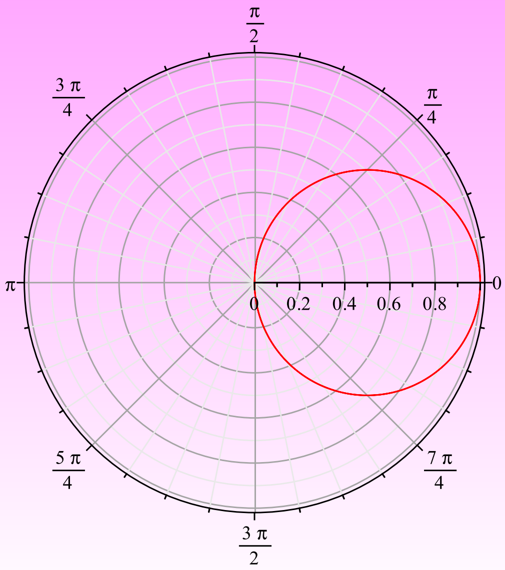
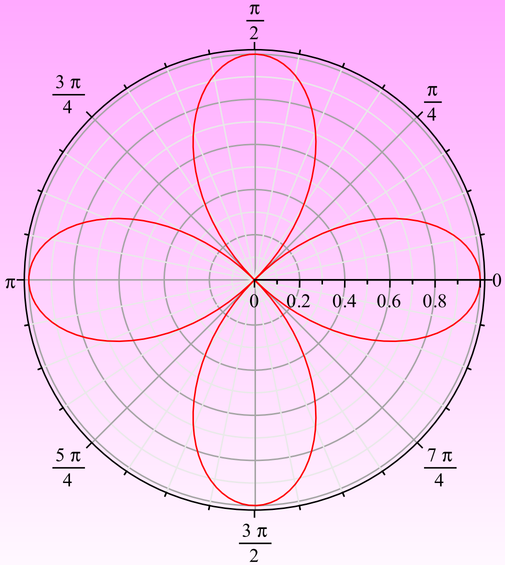
Graph of $r = \cos(\theta)$

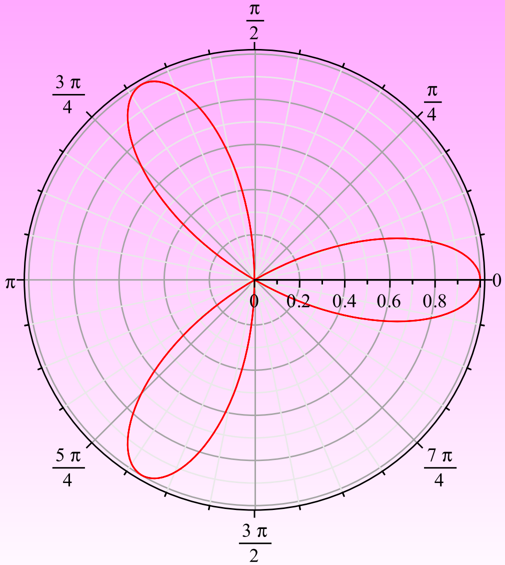


Graph of $r = \cos(2\theta)$



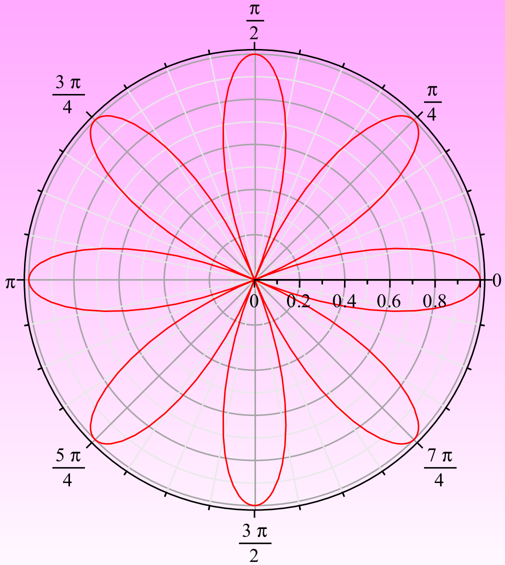
Graph of $r = \cos(3\theta)$

Graph of $r = \cos(3\theta)$



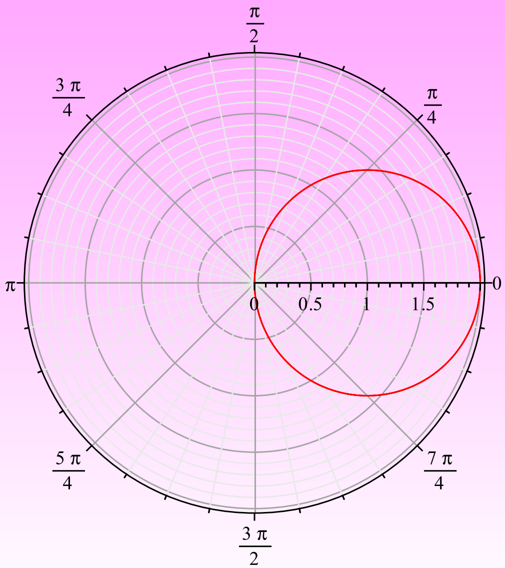
Graph of $r = \cos(4\theta)$

Graph of $r = \cos(4\theta)$



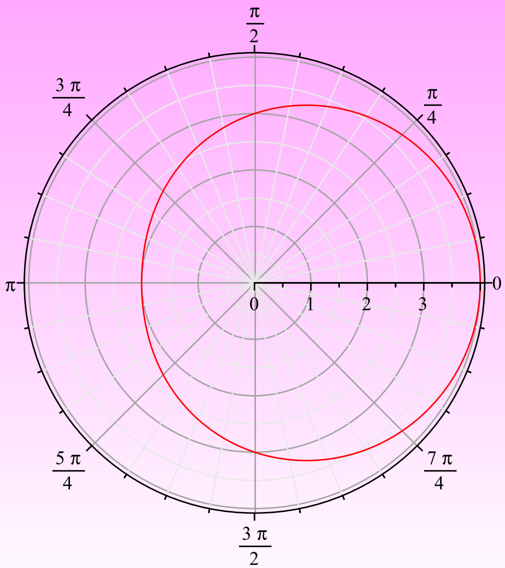
Graph of $r = 2\cos(\theta)$

Graph of $r = 2\cos(\theta)$



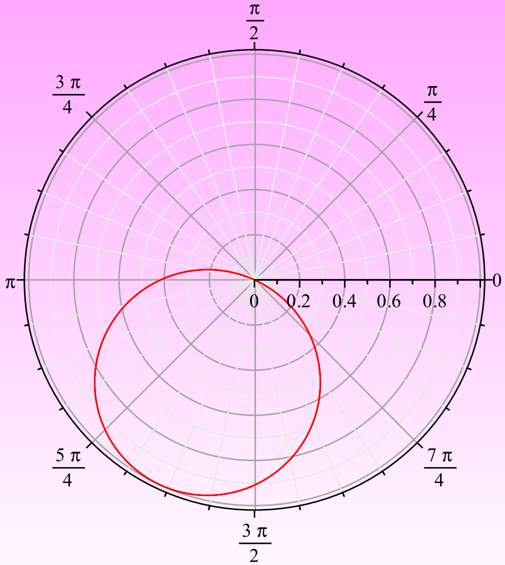
Graph of $r = \cos(\theta) + 3$

Graph of $r = \cos(\theta) + 3$



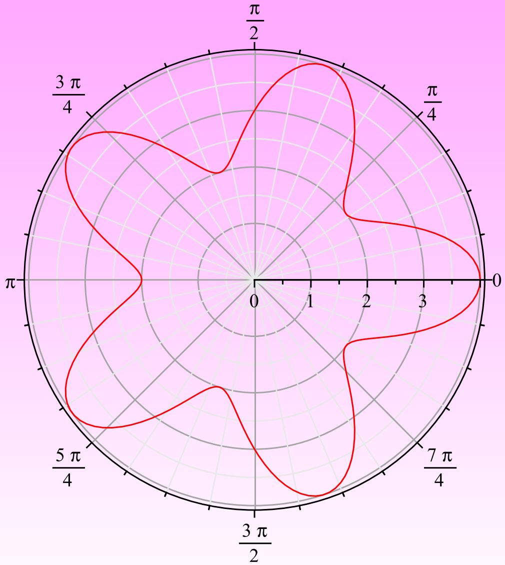
Graph of $r = \cos(\theta + 2)$

Graph of $r = \cos(\theta + 2)$

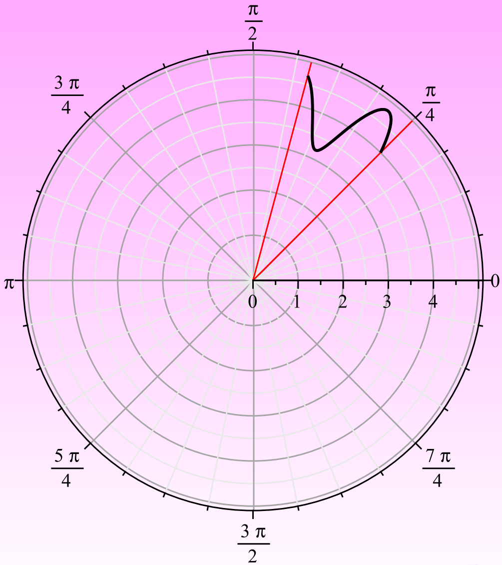


Graph of $r = 5\cos(\theta) + 3$

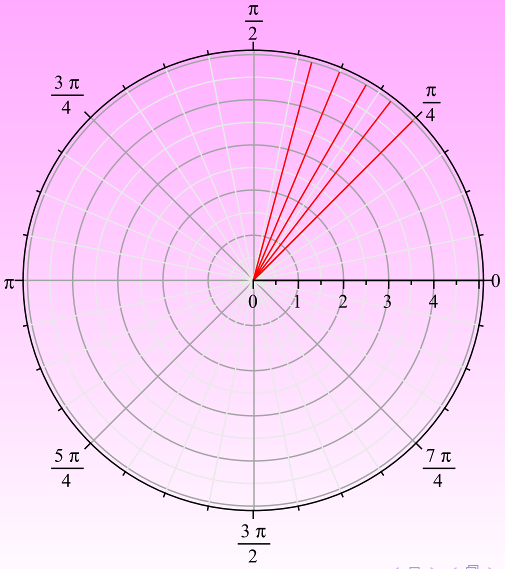
Graph of $r = 5\cos(\theta) + 3$



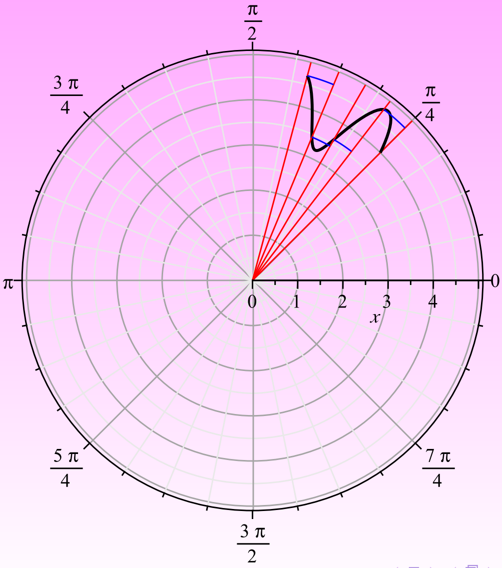
Area is "radial"



Fundamental Polar shapes are wedges not rectangles

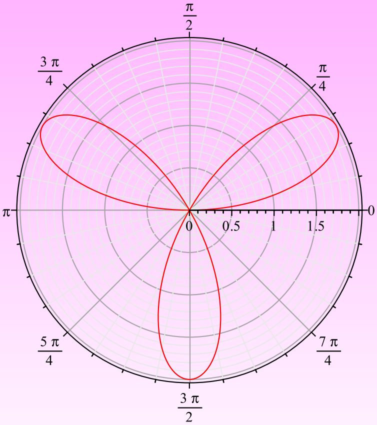


Approximate area with wedges:



Example:

Calculate the area inside one leaf of the three-leaf clover $r = 2\sin(3\theta)$.



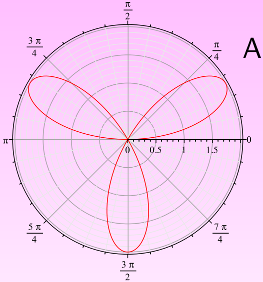
In Class Work

1. Sketch the graph of $r = 2\sin(3\theta)$, then find the area of one loop. NOTE: Sketching the graph, rather than using Maple, can end up saving you a little time, because you can see when you've completed one leaf, and know what to use for your upper and lower bounds of integration.
2. Sketch the area of $r = \sin(3\theta) + 4$, then find the area of the completed figure. Choose your interval of integration carefully.

Solutions

1. Sketch the graph of $r = 2\sin(3\theta)$, then find the area of one loop.

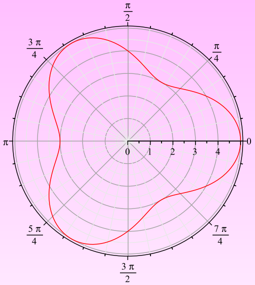
One loop: from when $2\sin(3\theta) = 0$ the first time to the next time, so from $\theta = 0$ to $\theta = \pi/3$



$$
\begin{aligned}
\text{Area} &= \frac{1}{2}\int_0^{\frac{\pi}{3}} \left[2\sin(3\theta)\right]^2 d\theta = 2\int_0^{\frac{\pi}{3}} \sin^2(3\theta)d\theta \\
&= 2\int_0^{\frac{\pi}{3}} \left[\frac{1}{2}\big(1-\cos(2(3\theta))\big)\right] d\theta \\
&= \int_0^{\frac{\pi}{3}} 1-\cos(6\theta)d\theta = \left[\theta - \frac{\sin(6\theta)}{6}\right]_0^{\frac{\pi}{2}} \\
&= \left[\left(\frac{\pi}{3} - \frac{\sin(6\pi/3)}{6}\right) - \left(0 - \frac{\sin(0)}{6}\right)\right] \\
&= \frac{\pi}{3}
\end{aligned}
$$

Solutions

2. Sketch the graph of $r = \sin(3\theta) + 4$, then find the area.
Took from $\theta = 0$ to $\theta = 2\pi$.



$$
\begin{aligned}
\text{Area} &= \frac{1}{2}\int_0^{2\pi} \big(\sin(3\theta)+4\big)^2 d\theta \\
&= \frac{1}{2}\int_0^{2\pi} \Big(\sin^2(3\theta) + 8\sin(3\theta) + 16\Big) d\theta \\
&= \frac{1}{2}\int_0^{2\pi} \left[\frac{1}{2}(1-\cos(2(3\theta))) + 8\sin(3\theta) + 16\right] d \\
&= \frac{1}{2}\int_0^{2\pi} \frac{1}{2} - \frac{\cos(6\theta)}{2} + 8\sin(3\theta) + 16 d\theta \\
&= \frac{1}{2}\left[\frac{33\theta}{2} - \frac{\sin(6\theta)}{12} - \frac{8\cos(3\theta)}{3}\right]_0^{2\pi} \\
&= \frac{1}{2}\left[(33\pi - 0 - \frac{8}{3}) - (0 - 0 - \frac{8}{3}))\right] = \frac{33\pi}{2}
\end{aligned}
$$