Recall:

If the region bounded by the function $y = f(x)$ and $y = 0$ for $a \leq x \leq b$ is rotated about the y-axis, then

$$V = \int_a^b 2\pi x f(x)\, dx.$$

In Class Work

Find the volume of each solid described below, using the shells.

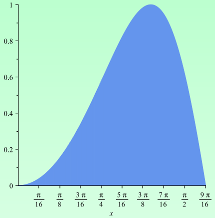
1. The solid formed when the region bounded by $y = \sin(x^2)$ and the x-axis for $0 \leq x \leq \sqrt{\pi}$ is rotated about the y-axis

2. The solid formed when the region bounded by by the parabola $y = -x^2 + 8x - 15$ and the x-axis is rotated about the line $x = 1$

3. The solid formed when the region bounded by $y = x$, $y = -x$, and $y = 1$ is rotated about the x-axis.

Solutions:

1. The solid formed when the region bounded by $y = \sin(x^2)$ and the x-axis for $0 \le x \le \sqrt{\pi}$ is rotated about the y-axis

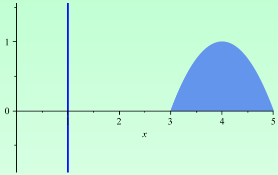


- Disks/washers isn't a good option
- **Use shells!**
- Shells around y-axis means dx
- radius=x, height=$\sin(x^2)$

$$\begin{aligned}
\text{Volume} &= \int_0^{\sqrt{\pi}} 2\pi x \sin(x^2)\, dx \\
u &= x^2, du = 2x\, dx \\
\text{Volume} &= \int_{x=0}^{x=\sqrt{\pi}} \pi \sin(u)\, du = -\pi \cos(x^2)\Big|_0^{\sqrt{\pi}} \\
&= -\pi\Big(\cos(\pi) - \cos(0)\Big) = 2\pi
\end{aligned}$$

Solutions:

2. The solid formed when the region bounded by by the parabola $y = -x^2 + 8x - 15$ and the x-axis is rotated about the line $x = 1$



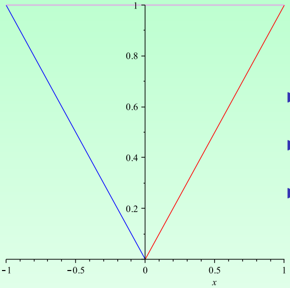
- Again use Shells
- Shells about vertical axis means dx
- height$=f(x) = -x^2 + 8x - 15$; radius$=x - 1$.
- Intersection points:

$$\begin{aligned} -x^2 + 8x - 15 = 0 \quad &\Rightarrow \quad -(x-5)(x-3) = 0 \\ &\Rightarrow \quad x = 3, x = 5 \end{aligned}$$

$$\begin{aligned} \text{Volume} &= \int_3^5 2\pi(x-1)(-x^2 + 8x - 15)\, dx \\ &= \ldots = 2\pi \int_3^5 -x^3 + 9x^2 - 23x + 15\, dx \\ &= 2\pi\left(-\frac{x^4}{4} + 3x^3 - \frac{23x^2}{2} + 15x\right)\Bigg|_3^5 = \ldots = 2\pi \cdot 4 = 8\pi \end{aligned}$$

Solutions

3. The solid formed when the region bounded by $y = x$, $y = -x$, and $y = 1$ is rotated about the x-axis.



- Washers would require two integrals. Use shells.
- Shells about x-axis means dy
- height=right-left=$y - (-y)$; radius=y.

$$\begin{aligned} \text{Volume} &= \int_0^1 2\pi(y)(2y)\,dy = 2\pi\int_0^1 2y^2\,dy \\ &= \frac{4\pi}{3}y^3\Big|_0^1 = \frac{4\pi}{3} \end{aligned}$$