In Class Work

1. Find two antiderivatives for $p(x) = 3x^5 + 7x^4 - \frac{3}{x} + \frac{11}{x^2}$ by hand.
2. Find the function $v(t)$ satisfying the conditions that $v'(t) = 2e^t - 3\cos(3t)$ and $v(0) = -3$.
3. Suppose that $f(x) = x^2 - 3e^{x^2} + 4$ and $A_f(x) = \int_{-5}^{x} f(t)\, dt$.
   (a) Is the graph of $f(x)$ increasing or decreasing at $x = -2$?
   (b) Is the graph of $f(x)$ concave up or concave down at $x = -2$?
   (c) Is the graph of $A_f(x)$ increasing or decreasing at $x = -2$?
   (d) Is the graph of $A_f(x)$ concave up or concave down at $x = -2$?
4. Find the signed areas given by the following integrals:
   (a) $\int_{1}^{4} \pi - x^{-3/2}\, dx$
   (b) $\int_{2}^{3} 6z^5 + \frac{5}{z^{10}}\, dz$

Solutions

1. Find two antiderivative for $p(x) = 3x^5 + 7x^4 - \frac{3}{x} + \frac{11}{x^2}$ by hand.

   One antiderivative: $P(x) = \frac{3}{6}x^6 + \frac{7}{5}x^5 - 3\ln|x| - \frac{11}{x}$

   No need for $+C$. Creates family of all *antiderivatives.*

   Adding any constant to $P(x)$ will give me a second antiderivative. For example, a second antiderivative is

   $$Q(x) = \frac{3}{6}x^6 + \frac{7}{5}x^5 - 3\ln|x| - \frac{11}{x} + e.$$

Solutions

2. Find the function $v(t)$ satisfying the conditions that $v'(t) = 2e^t - 3\cos(3t)$ and $v(0) = -3$.

Need to find: Specific antiderivative of $v'(t)$ that has $v(0) = 3$.

First, find any antiderivative:

- If you remember integration by substitution, use it.
- If you don't, guess, check and modify until you find an antiderivative. (We'll review substitution one day next week)

The family of antiderivatives is

$$v(t) = 2e^t - \sin(3t) + C$$

Need to find: The value of C that makes $v(0) = -3$:

$$-3 = v(0) = 2e^0 - \sin(0) + C = 2 + C \Longrightarrow C = -5.$$

Thus $v(t) = 2e^t - \sin(3t) - 5$.

Solutions:

3. Suppose that $f(x) = x^2 - 3e^{x^2} + 4$, and $A_f(x) = \int_{-5}^{x} f(t)\, dt$.
   (a) Is the graph of $f(x)$ increasing or decreasing at $x = -2$?
   $$f'(x) = 2x - 6xe^{x^2} \Rightarrow f'(-2) = -4 + 12e^4 > 0 \Rightarrow f \text{ is } \uparrow \text{ at } x = -2$$
   (b) Is the graph of $f(x)$ concave up or concave down at $x = -2$?
   $$f''(x) = 2 - 12x^2e^{x^2} - 6e^{x^2} \Rightarrow f''(-2) = 2 - (48 + 6)e^4 < 0.$$
   f is concave down at $x = -2$.
   (c) Is the graph of $A_f(x)$ increasing or decreasing at $x = -2$?
   Since A_f is an antiderivative of f, A_f' is f.
   $$A_f'(x) = f(x) = x^2 - 3e^{x^2} + 4 \Rightarrow A_f'(-2) = 4 - 3e^4 + 4 < 0.$$
   A_f is decreasing at $x = -2$.
   (d) Is the graph of $A_f(x)$ concave up or concave down at $x = -2$?
   Since $A_f''(x) = f'(x)$,
   $$A_f''(-2) = f'(-2) = -4 + 12e^4 > 0 \Rightarrow A_f \text{ concave up at } x = -2$$

Solutions:

4. Find the signed areas given by the following integrals:

   (a) $\int_1^4 \pi - x^{-3/2}\, dx$

   Use FTC, Part 1:

$$\begin{aligned} \int_1^4 \pi - x^{-3/2}\, dx &= \left[\pi x - \frac{x^{-1/2}}{-1/2}\right]_1^4 = \left[\pi x + \frac{2}{\sqrt{x}}\right]_1^4 \\ &= (4\pi + 1) - (\pi + 2) = 3\pi - 1 \end{aligned}$$

   (b) $\int_2^3 6z^5 + \frac{5}{z^{10}}\, dz$

$$\begin{aligned} \int_2^3 6z^5 + \frac{5}{z^{10}}\, dz &= \left[z^6 - \frac{5}{9}z^{-9}\right]_2^3 = \left(3^6 - \frac{5}{9 \cdot 3^9}\right) - \left(2^6 - \frac{5}{9 \cdot 2^9}\right) \\ &\approx 949.44 \end{aligned}$$