Using Polynomials to Approximate Other Functions

By insisting that the first six derivatives match at $x = 0$, we can create a 6th degree polynomial $P_6(x)$ that approximates $\cos(x)$ fairly well.
That is, by forcing $P_6(x)$ to satisfy

$$\begin{aligned} P_6(0) &= \cos(0) & P_6^{(4)}(0) &= \cos(0) \\ P_6'(0) &= -\sin(0) & P_6^{(5)}(0) &= -\sin(0) \\ P_6''(0) &= -\cos(0) & P_6^{(6)}(0) &= -\cos(0) \\ P_6'''(0) &= \sin(0) \end{aligned}$$

We can find $P_6(x)$ so that

$$P_6(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} \text{ approximates } \cos(x) \text{ near } 0.$$

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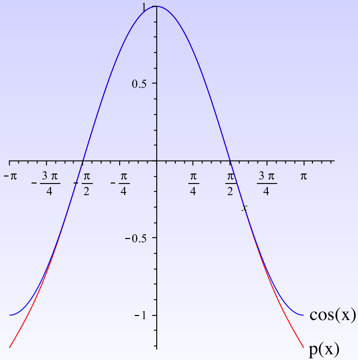
(For those who've seen them before, this is a Taylor (or MacLaurin) polynomial.)



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We can use these *Taylor Polynomial* approximations to approximate

- The value of a more complicated function at a particular point
- The derivative of a more complicated function at a point
- The integral of a more complicated function at a point

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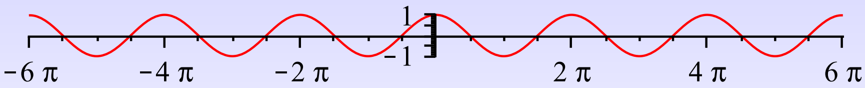
The more derivatives *at* $x = 0$ we match (the higher n is), the better $P_n(x)$ does at approximating $\cos(x)$ *away* from $x = 0$.

The following is a graph of $P_{50}(x)$:



Goal:

Work toward formalizing the idea of an **infinite sum**, so that we can use "infinite degree polynomials" to replace transcendental functions (because they're equal).