Fun Fact: The Set Up

The set of a game show has three closed doors. Behind one of these doors is a car; behind the other two are goats. The contestant does not know where the car is, but the host does.

- The contestant picks a door.
- The host opens one of the two *remaining* doors, one he **knows** doesn't hide the car, showing one of the two goats. (If the contestant has chosen the correct door, the host is equally likely to open either of the two remaining doors.)
- After the host has shown a goat, the contestant is always given the option to switch doors.

Question: What is the probability that the contestant will win the car if she stays with her first choice? Does that probability change if she changes to the remaining door?

Fun Fact

The contestant should switch doors after being shown the goat.

The probability of winning the car if she doesn't switch doors is only $1/3$, while the probability of winning the car if she does switch doors is $2/3$!

Fun Fact

Claim: The probability of winning the car if she doesn't switch doors is only $1/3$, while the probability of winning the car if she does switch doors is $2/3$!

Why?

- When the contestant first chooses a door, it is equally likely that the car is behind any one of the three doors.
- Thus:
  P(car behind chosen door)=$1/3$
  P(not behind chosen door)=$2/3$
- Host opens one of the two doors the contestant *didn't* choose.
- But that doesn't change the probabilities of where the car is!

Goals:

Be able to :

1. determine whether a series $\sum a_k$ converges or diverges.
2. If it converges, find the limit (that is, the value of the series) exactly, if possible.
3. If it converges but we can't find the limit exactly, be able to approximate it.

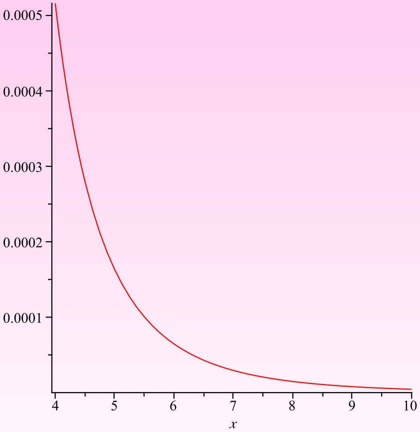
How can we tell whether a series converges or diverges?

What we have so far:

1. Is it a geometric series? If so, it converges if $|r| < 1$.
2. Does the nth term test for divergence apply?

We need more methods to test for convergence!

Graph of $a(x) = \dfrac{2x^3}{(2x^4 - 14)^2}$ on $[4, \infty)$



In Class Work

1. Use the integral test to determine whether the following series converge or diverge.

   (a) $\displaystyle\sum_{j=3}^{\infty} \frac{18j^2}{3j^3+1}$

   (b) $\displaystyle\sum_{k=1}^{\infty} \frac{k}{e^{k^2}}$

2. Let $S = \displaystyle\sum_{k=1}^{\infty} \frac{k}{e^{k^2}}$. Use the integral test to find lower and upper limits for the value of S.

Solutions:

Use the integral test to determine whether the following series converge or diverge.

1(a) $\displaystyle\sum_{j=3}^{\infty} \frac{18j^2}{3j^3+1}$

The graph of $f(x) = \dfrac{18x^2}{3x^3+1}$ shows that this function is continuous, positive, and decreasing, so the integral test applies.

Integral Test $\Longrightarrow \displaystyle\sum_{j=3}^{\infty} \frac{18j^2}{3j^3+1}$ will do whatever $\displaystyle\int_3^{\infty} \frac{18x^2}{3x^3+1}\,dx$ does.

$$\int_3^{\infty} \frac{18x^2}{3x^3+1}\,dx = 2\int_{28}^{\infty} \frac{1}{u}\,du = 2\lim_{R\to\infty}\int_{28}^{R} \frac{1}{u}\,du$$

We know this integral diverges, so the series diverges as well.

Solutions:

1(b) $\displaystyle\sum_{k=1}^{\infty} \frac{k}{e^{k^2}}$

Again, the graph shows that the function is continuous, positive, and decreasing from about $x = 1$ on, so the integral test applies.

Integral Test $\Longrightarrow \displaystyle\sum_{k=1}^{\infty} \frac{k}{e^{k^2}}$ will do whatever $\displaystyle\int_1^{\infty} xe^{-x^2}\, dx$ does.

$$\int_1^{\infty} xe^{-x^2}\, dx = \int_{-1}^{-\infty} -\frac{1}{2}e^u\, du = -\frac{1}{2}\lim_{R\to\infty} e^u\Big|_{-1}^{-R} = \frac{1}{2e}.$$

Since this integral converges, so does the series (although it does not converge to $\frac{1}{2e}$)

Solutions:

Let $S = \sum_{k=1}^{\infty} \frac{k}{e^{k^2}}$.

2. Use the integral test to find lower and upper limits for the value of S.

   Integral test $\implies \int_1^{\infty} a(x)\ dx \leq \sum_{k=1}^{\infty} a_k \leq 1 + \int_0^{\infty} a(x)\ dx$.

   Since we found that $\int_1^{\infty} xe^{-x^2}\ dx = \frac{1}{2e}$,

$$\begin{aligned}
\int_1^{\infty} xe^{-x^2}\ dx \quad &\leq S \leq \quad a_1 + \int_1^{\infty} xe^{-x^2}\ dx \\
\frac{1}{2e} \quad &\leq S \leq \quad \frac{1}{e} + \frac{1}{2e} = \frac{3}{2e} \\
0.184 \quad &\leq S \leq \quad 0.552
\end{aligned}$$