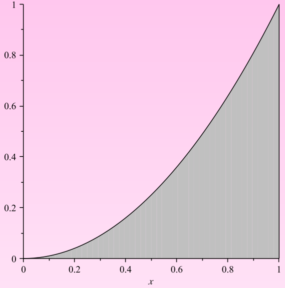
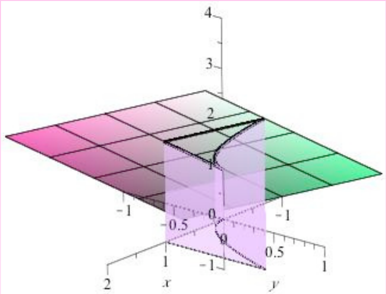
1. Find the signed volume between the surface $z = 1 + x + y$ and the region R in the xy-plane bounded by the graphs $x = 1, y = 0, y = x^2$.





Looking at the region R shown on the left, we can see that we can either say that

$$0 \leq x \leq 1 \text{ and } 0 \leq y \leq x^2$$

or

$$0 \leq y \leq 1 \text{ and } \sqrt{y} \leq x \leq 1.$$

I will choose to use $0 \leq x \leq 1$ and $0 \leq y \leq x^2$

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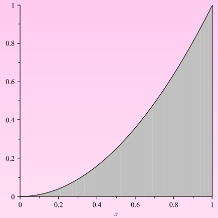

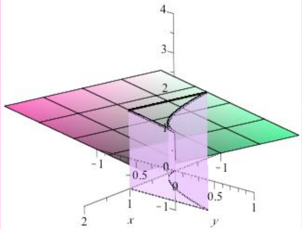

Choosing to use $0 \le x \le 1$ and $0 \le y \le x^2$

Thus by Fubini's Theorem,

$$V = \iint_R 1 + x + y \; dA = \int_0^1 \left(\int_0^{x^2} 1 + x + y \; dy \right) \; dx.$$

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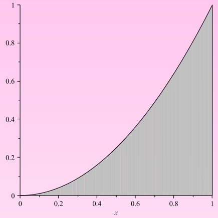
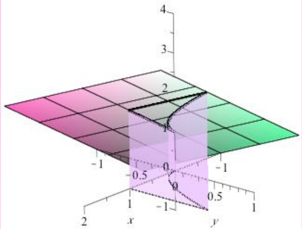
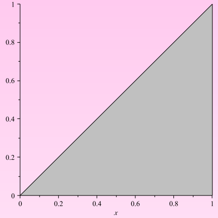


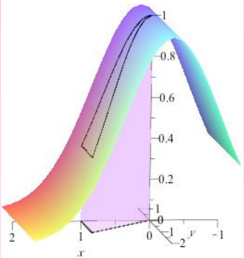


Thus by Fubini's Theorem,

$$\begin{aligned} V &= \iint_R 1 + x + y \, dA = \int_0^1 \left(\int_0^{x^2} 1 + x + y \, dy \right) dx \\ &= \int_0^1 \left(y + xy + \frac{y^2}{2} \Big|_0^{x^2} \right) dx \\ &= \int_0^1 x^2 + x^3 + \frac{1}{2} x^4 \, dx = \ldots = \frac{41}{60} \end{aligned}$$

2. Find the volume below the surface $z = e^{-x^2}$ and above the triangle R in the xy-plane bounded by the x-axis, the line $x = 1$, and the line $y = x$.





Looking at the region R shown on the left, we see that we can write the region in two ways again:

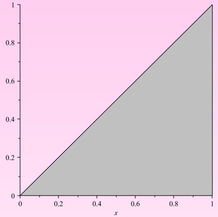
$$0 \leq x \leq 1 \text{ and } 0 \leq y \leq x$$

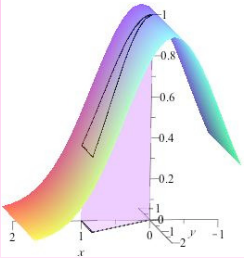
or

$$0 \leq y \leq 1 \text{ and } y \leq x \leq 1.$$

I will choose to use $0 \leq x \leq 1$ and $0 \leq y \leq x$

2. Find the volume below the surface $z = e^{-x^2}$ and above the triangle R in the xy-plane bounded by the x-axis, the line $x = 1$, and the line $y = x$.





Choosing to use $0 \le x \le 1$ and $0 \le y \le x$

Thus by Fubini's Theorem,

$$V = \iint_R e^{-x^2} \, dA = \int_0^1 \left(\int_0^x e^{-x^2} \, dy \right) \, dx$$

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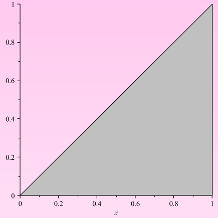
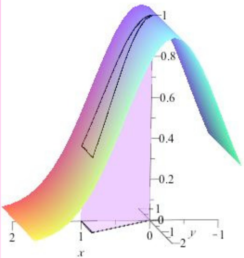




Thus by Fubini's Theorem,

$$
\begin{aligned}
V &= \iint_R e^{-x^2}\, dA = \int_0^1 \left(\int_0^x e^{-x^2}\, dy \right)\, dx \\
&= \int_0^1 \left(ye^{-x^2} \Big|_0^x \right)\, dx \\
&= \int_0^1 xe^{-x^2}\, dx \overset{u-sub}{=} \ldots = -\frac{1}{2}\left(\frac{1}{e} - 1\right) = -\frac{1}{2e} + \frac{1}{2}
\end{aligned}
$$

3.(a)

Try to evaluate $\int_0^\pi \int_x^\pi \frac{\sin(y)}{y}\, dy\, dx$ as it's written. What happens?

$$\int_0^\pi \int_x^\pi \frac{\sin(y)}{y}\, dy\, dx = \int_0^\pi \int_x^\pi \frac{1}{y} \sin(y)\, dy\, dx$$

I can already see that neither substitution nor integration by parts is going to work wonders on this integral.

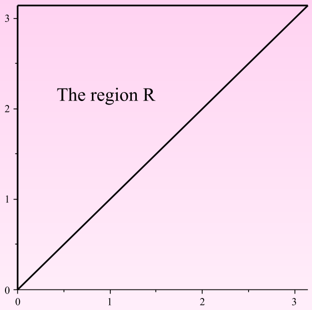
When I try to do the inner integral on Maple, I get some function called "Si(y)", which I've never heard of ... or at least, it rings no bells.

3(b) Sketch the region we're integrating over.

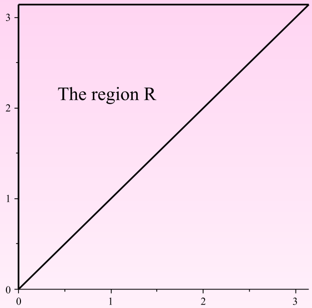
In order to sketch the region, I need to look at what intervals I'm integrating over.

Looking at the integral $\int_0^\pi \left(\int_x^\pi \frac{\sin(y)}{y}\, dy \right) dx$, we see that $0 \leq x \leq \pi$ and $x \leq y \leq \pi$.

In other words, as x goes from 0 to π, y goes from the diagonal lines $y = x$ up to the horizontal line $y = \pi$.



3(c) Reverse the order of integration (using the sketch you developed in (b)), and try to evaluate the integral. Is this way more effective than the first?



Looking at the region R, I can see that we can also say that

$$0 \leq y \leq \pi \text{ and } 0 \leq x \leq y.$$

We can thus rewrite the integral:

$$\int_0^{\pi} \left(\int_0^{y} \frac{\sin(y)}{y} \, dx \right) \, dy.$$

Because this time we're integrating with respect to x first, and because our integrand is constant with respect to x, this is suddenly much easier!

3(c) Reverse the order of integration (using the sketch you developed in (b)), and try to evaluate the integral. Is this way more effective than the first?

We have just found that

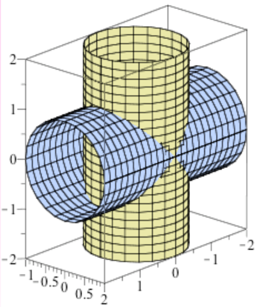
$$\int_0^\pi \int_0^y \frac{\sin(y)}{y}\, dy\, dx = \int_0^\pi \left(\int_0^y \frac{\sin(y)}{y}\, dx\right)\, dy.$$

The integral on the left is a very difficult integral for us. As for the integral on the *right* ...

$$\begin{aligned}
\int_0^\pi \left(\int_0^y \frac{\sin(y)}{y}\, dx\right)\, dy &= \int_0^\pi \left(\frac{\sin(y)}{y}\int_0^y 1\, dx\right)\, dy \\
&= \int_0^\pi \left(\frac{\sin(y)}{y}(x)\Big|_0^y\right)\, dy \\
&= \int_0^\pi \frac{\sin(y)}{y}(y-0)\, dy = \int_0^\pi \sin(y)\, dy \\
&= -\cos(y)\Big|_0^\pi = -(-1)-(-1) = 2
\end{aligned}$$

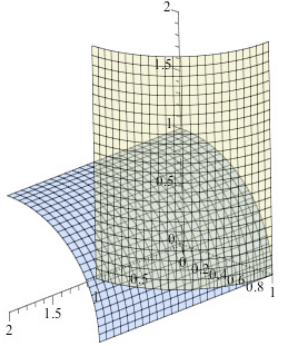
4. Find the volume of portion of the solid bounded by the cylinders $x^2 + y^2 = 1$ and $y^2 + z^2 = 1$ that lies in the first octant.

$x^2 + y^2 = 1$ is a cylinder of radius 1 extending upwards along the z-axis (in pale yellow), while $y^2 + z^2 = 1$ is a cylinder of radius 1 extending along the x-axis (in skyblue).



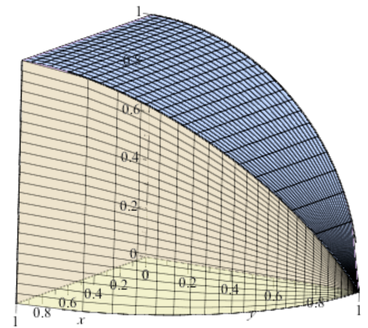
4. Find the volume of portion of the solid bounded by the cylinders $x^2 + y^2 = 1$ and $y^2 + z^2 = 1$ that lies in the first octant.

We only want to consider the portion that lies in the first octant; that is, where $x \geq 0$, $y \geq 0$, and $z \geq 0$:

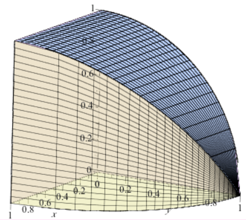


4. Find the volume of portion of the solid bounded by the cylinders $x^2 + y^2 = 1$ and $y^2 + z^2 = 1$ that lies in the first octant.

We want the portion that's enclosed by the *xz*-plane and the *yz*-plane as two sides, the portion of the top-half of the cylinder $y^2 + z^2 = 1$ that lies above the quarter of the unit circle that lies in the first quadrant of the *xy*-plane, and that has as its third "side" the portion of the vertical cylinder that goes from the *xy*-plane to this top portion.



4. Find the volume of portion of the solid bounded by the cylinders $x^2 + y^2 = 1$ and $y^2 + z^2 = 1$ that lies in the first octant.



Our solid lies under the top half of $y^2 + z^2 = 1$, over the region that is the quarter unit-circle that lies in the first quadrant of the *xy*-plane.

Thus the function we're integrating is

$$z = \sqrt{1 - y^2},$$

and the region we're integrating over is

$$R : 0 \leq x \leq 1, 0 \leq y \leq \sqrt{1 - x^2} \;\; or \;\; R : 0 \leq y \leq 1, 0 \leq x \leq \sqrt{1 - y^2}.$$

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Thus the volume is given by

$$V = \int_0^1 \left(\int_0^{\sqrt{1-x^2}} \sqrt{1 - y^2} \, dy \right) \, dx$$

or

$$V = \int_0^1 \left(\int_0^{\sqrt{1-y^2}} \sqrt{1 - y^2} \, dx \right) \, dy.$$

4. Find the volume of portion of the solid bounded by the cylinders $x^2 + y^2 = 1$ and $y^2 + z^2 = 1$ that lies in the first octant.

$$V = \int_0^1 \int_0^{\sqrt{1-x^2}} \sqrt{1-y^2}\, dy\, dx \text{ or } V = \int_0^1 \int_0^{\sqrt{1-y^2}} \sqrt{1-y^2}\, dx\, dy.$$

Which would we rather do?

4. Find the volume of portion of the solid bounded by the cylinders $x^2 + y^2 = 1$ and $y^2 + z^2 = 1$ that lies in the first octant.

$$
\begin{aligned}
V &= \int_0^1 \int_0^{\sqrt{1-y^2}} \sqrt{1-y^2}\, dx\, dy \\
&= \int_0^1 x\sqrt{1-y^2}\Big|_0^{\sqrt{1-y^2}}\, dy \\
&= \int_0^1 1 - y^2\, dy \\
&= y - \frac{1}{3}y^3\Big|_0^1 \\
&= 1 - \frac{1}{3} = \frac{2}{3}
\end{aligned}
$$