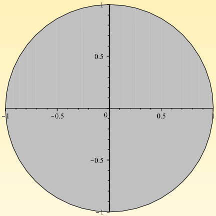
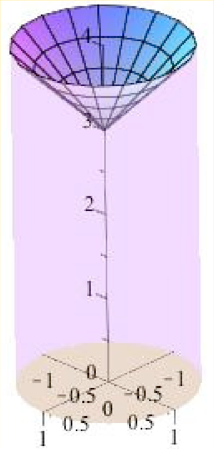
Example:

Find the volume below $f(x, y) = \sqrt{x^2 + y^2} + 3$ and above the unit circle in the xy-plane.





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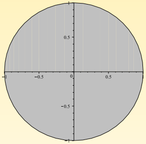
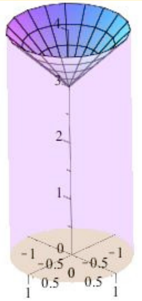
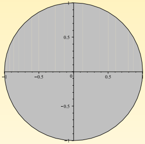


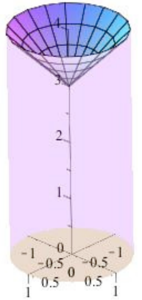


$$
\begin{aligned}
V &= \iint_R \sqrt{x^2 + y^2} + 3 \, dA \\
&= \int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \sqrt{x^2 + y^2} + 3 \; dy \; dx
\end{aligned}
$$

Example:

Find the volume below $f(x,y) = \sqrt{x^2+y^2} + 3$ and above the unit circle in the xy-plane.





$$\begin{aligned} V &= \iint_R \sqrt{x^2+y^2} + 3 \, dA \\ &= \int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \sqrt{x^2+y^2} + 3 \; dy \; dx \end{aligned}$$

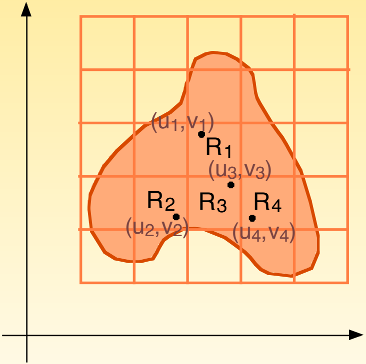
Not a very pretty double integral!

Recall: Converting between polar and rectangular coordinates:

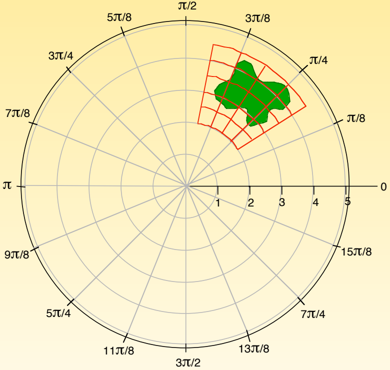
- $r^2 = x^2 + y^2$
- $x = r\cos(\theta)$
- $y = r\sin(\theta)$

"Partitioning" our region R:

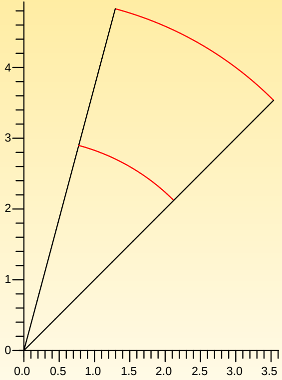
In rectangular coordinates



In polar coordinates

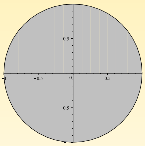


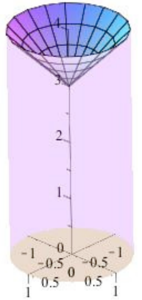
An elementary polar region, i.e. a "polar rectangle"



Example: where we left off

Find the volume below $f(x,y) = \sqrt{x^2 + y^2} + 3$ and above the unit circle in the xy-plane.





$$R : 0 \le r \le 1 \qquad 0 \le \theta \le 2\pi$$

$$V = \iint_R r + 3\, dA$$

In Class Work

Find the volume of the solid that lies under the upper hemisphere $z = \sqrt{25 - x^2 - y^2}$, above the xy-plane, and inside the cylinder $x^2 + y^2 = 5x$.

Note: $x^2 + y^2 = 5x$ *is* a circle in 2-space, as you could tell by completing the square. But rather than fussing with that, the presence of the $x^2 + y^2$ piece in both the surface and the region we're integrating over gives us the clue that polar is the way to go! So convert everything ...