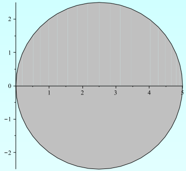
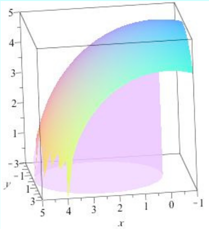
Find the volume of the solid that lies under the upper hemisphere $z = \sqrt{25 - x^2 - y^2}$, above the $xy$-plane, and inside the cylinder $x^2 + y^2 = 5x$.





- **Convert the function to polar coordinates:**
  Since $r^2 = x^2 + y^2$, $z = g(r, \theta) = \sqrt{25 - r^2}$.

Find volume between $z = \sqrt{25 - x^2 - y^2}$, and $xy$-plane above $R : x^2 + y^2 = 5x$.

- In polar coordinates, $f$ is $z = g(r, \theta) = \sqrt{25 - r^2}$.
- **Convert the region $R$ to polar coordinates:**

Since $r^2 = x^2 + y^2$ and $x = r\cos(\theta)$,

$$r^2 = 5r\cos(\theta) \Rightarrow r(r - 5\cos(\theta)) = 0 \qquad r = 5\cos(\theta).$$

*(Why? Because the function isn't $r = 0$, since that's just a point.)*

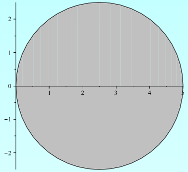
As we know, this is a circle (not centered at the origin).

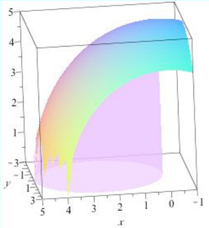
To fill it in, we need for $r$ to go from 0 to $5\cos(\theta)$.

What about $\theta$? We get the whole circle just by going from 0 to $\pi$ – going around twice might double the volume, *or* might make it be 0.

Thus $R : 0 \leq r \leq 5\cos(\theta), 0 \leq \theta \leq \pi$.

Find volume between $z = \sqrt{25 - x^2 - y^2}$, and $xy$-plane above $R : x^2 + y^2 = 5x$.





- In polar coordinates, $f$ is $z = g(r, \theta) = \sqrt{25 - r^2}$.
- In polar coordinates, $R$ is described by : $0 \le r \le 5\cos(\theta), 0 \le \theta \le \pi$.
- **Set up the integral**

$$V = \iint_R \sqrt{25 - x^2 - y^2}\, dA$$

$$= \int_0^{\pi} \int_0^{5\cos(\theta)} \sqrt{25 - r^2}(r\, dr\, d\theta)$$

Find volume between $z = \sqrt{25 - x^2 - y^2}$, and $xy$-plane above $R : x^2 + y^2 = 5x$.

- In polar coordinates, $f$ is $z = g(r, \theta) = \sqrt{25 - r^2}$.
- In polar coordinates, $R$ is described by : $0 \leq r \leq 5\cos(\theta), 0 \leq \theta \leq \pi$.
- The volume is given by:

$$V = \iint_R \sqrt{25 - x^2 - y^2}\, dA = \int_0^{\pi} \int_0^{5\cos(\theta)} \sqrt{25 - r^2}(r\, dr\, d\theta)$$

- **Find the integral.** Let $u = 25 - r^2$, so $-\frac{1}{2}\, du = r\, dr$. Then

$$V = -\frac{1}{2} \int_0^{\pi} \frac{2}{3}(25 - r^2)^{3/2} \Big|_0^{5\cos\theta} \, d\theta.$$

Find volume between $z = \sqrt{25 - x^2 - y^2}$, and $xy$-plane above $R : x^2 + y^2 = 5x$.

$$
\begin{aligned}
V &= -\frac{1}{2}\int_0^{\pi} \frac{2}{3}(25 - r^2)^{3/2}\Big|_0^{5\cos\theta} \ d\theta \\
&= -\frac{1}{3}\int_0^{\pi} (25 - (5\cos\theta)^2)^{3/2} - 125 \ d\theta \\
&= -\frac{1}{3}\int_0^{\pi} (5)^3 \sin^3\theta - 125 \ d\theta \\
&= -\frac{125}{3}\int_0^{\pi} (1 - \cos^2\theta)\sin\theta - 1 \ d\theta \\
&= \frac{125}{3}\cos\theta - \frac{1}{3}\cos^3(\theta)\Big|_0^{\pi} + \frac{125 * (\pi)}{3} \\
&= \frac{125}{3}[(-1 + \frac{1}{3}) - (1 - \frac{1}{3})] + \frac{125\pi}{3} \\
&= \frac{125}{3}(\frac{-4}{3}) + \frac{125\pi}{3} = -\frac{500}{9} + \frac{125\pi}{3}
\end{aligned}
$$