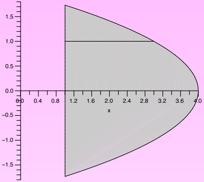
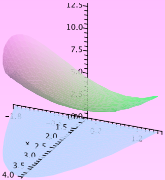
1. Find the surface area of the portion of $z = x^2 + y^2$ between $x = 4 - y^2$ and $x = 1$



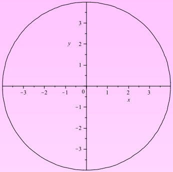


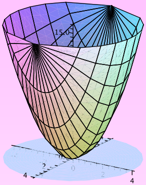
$$
\begin{aligned}
SA &= \iint_R \sqrt{f_x(x,y)^2 + f_y(x,y)^2 + 1}\, dA \\
&= \int_{-\sqrt{3}}^{\sqrt{3}} \int_1^{4-y^2} \sqrt{f_x(x,y)^2 + f_y(x,y)^2 + 1}\, dx\, dy \\
&= \int_{-\sqrt{3}}^{\sqrt{3}} \int_1^{4-y^2} \sqrt{4x^2 + 4y^2 + 1}\, dx\, dy
\end{aligned}
$$

You can't do this, and neither can Maple, exactly- so approximate!

$$SA \approx 33.47.$$

2. Find the surface area of the portion of $z = x^2 + y^2$ inside the cylinder $x^2 + y^2 = 16$ *by hand.*

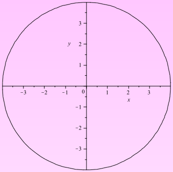


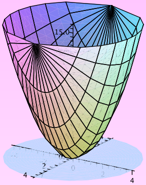


$$\begin{aligned} SA &= \iint_R \sqrt{f_x(x,y)^2 + f_y(x,y)^2 + 1}\, dA \\ &= \int_{-4}^{4} \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \sqrt{4x^2 + 4y^2 + 1}\, dy\, dx \end{aligned}$$

Nasty looking!

2. Find the surface area of the portion of $z = x^2 + y^2$ inside the cylinder $x^2 + y^2 = 16$ *by hand.*





Because both the region we're integrating over and the surface are circular in nature, our double integral is screaming to be converted to polar!

- Region R: circle of radius 4 centered at the origin, so $0 \leq r \leq 4$ and $0 \leq \theta \leq 2\pi$.
- Remember that $x^2 + y^2 = r^2$, and $dx\ dy = r\ dr\ d\theta$.

2. Find the surface area of the portion of $z = x^2 + y^2$ inside the cylinder $x^2 + y^2 = 16$ *by hand.*

$$SA = \int_{-4}^{4} \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \sqrt{4x^2 + 4y^2 + 1}\, dy\, dx$$

- Region R: $0 \le r \le 4$ and $0 \le \theta \le 2\pi$.
- $x^2 + y^2 = r^2$, and $dx\, dy = r\, dr\, d\theta$.

$$\begin{aligned} SA &= \int_{-4}^{4} \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \sqrt{4x^2 + 4y^2 + 1}\, dy\, dx \\ &= \int_{0}^{2\pi} \int_{0}^{4} (\sqrt{4r^2 + 1})r\, dr\, d\theta \end{aligned}$$

Much better!

2. Find the surface area of the portion of $z = x^2 + y^2$ inside the cylinder $x^2 + y^2 = 16$ *by hand.*

$$
\begin{aligned}
SA &= \int_{-4}^{4} \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \sqrt{4x^2 + 4y^2 + 1}\, dy\, dx \\
&= \int_{0}^{2\pi} \int_{0}^{4} (\sqrt{4r^2 + 1}) r\, dr\, d\theta
\end{aligned}
$$

Let $u = 4r^2 + 1$.

Then $du = 8r\, dr$, and $r = 0 \Rightarrow u = 1; r = 4 \Rightarrow u = 65$

$$
\begin{aligned}
&= \frac{1}{8} \int_{0}^{2\pi} \int_{1}^{65} u^{1/2}\, du\, d\theta \\
&= \frac{1}{12} \int_{0}^{2\pi} u^{3/2} \Big|_{1}^{65}\, d\theta = \frac{1}{12} \int_{0}^{2\pi} 65^{3/2} - 1\, d\theta \\
&= \frac{\pi}{6}(65^{3/2} - 1) \approx 273.87.
\end{aligned}
$$