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- **Surface Area**$= \iint_R \sqrt{f_x(x,y)^2 + f_y(x,y)^2 + 1}$

One Double Integral, Many Stories

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The same integral can have different interpretations depending upon the context.

Definition:

- A **vector field** in the plane is a function $\overrightarrow{\mathbf{F}} : \mathbb{R}^2 \to V^2$ (where V^2 is the set of two-dimensional vectors).

  We write
  $$\overrightarrow{\mathbf{F}}(x, y) = \langle f_1(x, y), f_2(x, y) \rangle$$
  for scalar functions $f_1, f_2 : \mathbb{R}^2 \to \mathbb{R}$.

- A **vector field** in space is a function $\overrightarrow{\mathbf{F}} : \mathbb{R}^3 \to V^3$.

  We write
  $$\overrightarrow{\mathbf{F}}(x, y, z) = \langle f_1(x, y, z), f_2(x, y, z), f_3(x, y, z) \rangle$$
  for scalar functions $f_1, f_2, f_3 : \mathbb{R}^3 \to \mathbb{R}$.

In Class Work

1. Show that

   $$\overrightarrow{\mathbf{F}}(x, y) = < \cos(y) - 2 + y^2 e^{xy^2}, \cos(y) - x\sin(y) + 3y^2 + 2xye^{xy^2} >$$

   is a conservative vector field by finding its potential function $f(x, y)$.

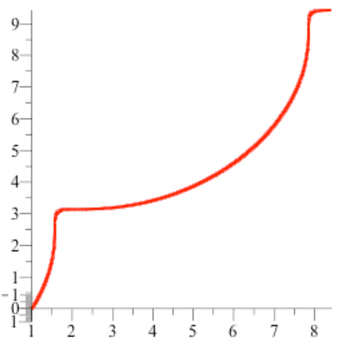
2. Show that

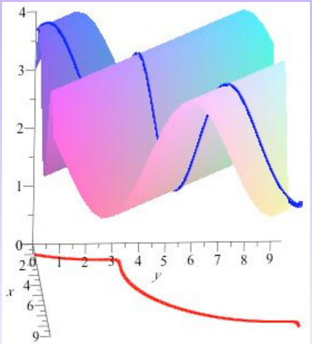
   $$\overrightarrow{\mathbf{F}}(x, y) = < 2y + 2xy^2, 2x + 2y >$$

   is not a conservative vector field.

Integrating over a curve $\mathcal{C}$

Suppose we have a curve $\mathcal{C}$ in $\mathbb{R}^2$ and a surface $z = f(x, y)$.





Integrating over a curve $\mathcal{C}$

We want to find the area of the vertical surface that we create if we go straight up from the curve $\mathcal{C}$ to the surface.