Recall from Calc 1:

For $g(x)$, if $g'(a) = 0$, then the 2nd Derivative Test gives

$$\begin{array}{lcl} g'(a) > 0 & \Longrightarrow & x = a \textit{ is a minimum} \\ g'(a) < 0 & \Longrightarrow & x = a \textit{ is a maximum} \\ g'(a) = 0 & \Longrightarrow & \textit{the test is inconclusive} \end{array}$$

Is there an analogous result for $f(x, y)$?

Example:

Let $f(x,y) = x^2 + 3xy + 2y^2 - 9x - 11y$.

$$f_x(x,y) = 2x + 3y - 9 \implies f_x(-3,5) = -6 + 15 - 9 = 0$$
$$f_y(x,y) = 3x + 4y - 11 \implies f_y(-3,5) = -9 + 20 - 11 = 0$$

$$f_{xx}(x,y) = 2 \implies f_{xx}(-3,5) > 0 \qquad f_{yy}(x,y) = 4 \implies f_{yy}(-3,5) > 0$$

Example:

Let $f(x,y) = x^2 + 3xy + 2y^2 - 9x - 11y$.

$$f_x(x,y) = 2x + 3y - 9 \implies f_x(-3,5) = -6 + 15 - 9 = 0$$
$$f_y(x,y) = 3x + 4y - 11 \implies f_y(-3,5) = -9 + 20 - 11 = 0$$

$$f_{xx}(x,y) = 2 \implies f_{xx}(-3,5) > 0 \qquad f_{yy}(x,y) = 4 \implies f_{yy}(-3,5) > 0$$

Thus at $(-3,5)$, f is flat in both the x and y directions, *and* is concave up in both the x and y directions ...

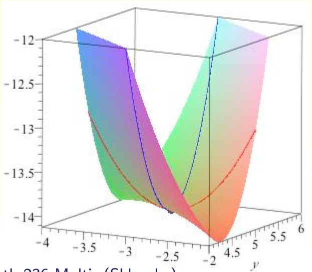
Example:

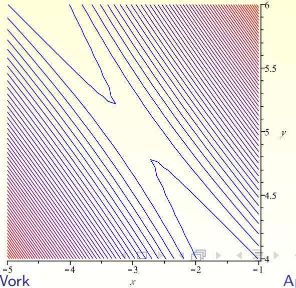
Let $f(x,y) = x^2 + 3xy + 2y^2 - 9x - 11y$.

$$f_x(x,y) = 2x + 3y - 9 \implies f_x(-3,5) = -6 + 15 - 9 = 0$$
$$f_y(x,y) = 3x + 4y - 11 \implies f_y(-3,5) = -9 + 20 - 11 = 0$$

$f_{xx}(x,y) = 2 \implies f_{xx}(-3,5) > 0$ $\quad$ $f_{yy}(x,y) = 4 \implies f_{yy}(-3,5) > 0$

Thus at $(-3,5)$, f is flat in both the x and y directions, *and* is concave up in both the x and y directions ... *but* f does **not** have a minimum there:





1. Locate all critical points and classify them using the 2nd Derivatives Test:

   (a) $f(x, y) = x^3 - 3xy + y^3$

   (b) $f(x, y) = y^2 + x^2y + x^2 - 2y$

2. Let $f(x, y) = 4xy - x^3 - 2y^2$

   (a) Find and classify all critical points of f.

   (b) Find the maximum value of $f(x, y)$ on the unit disk $\{(x, y) \mid x^2 + y^2 \leq 1\}$.