Recall:

Green's Theorem: Let $\mathcal{C}$ be a piecewise-smooth, simple closed curve in $\mathbb{R}^2$ with positive orientation that encloses the region R. Suppose that $\overrightarrow{\mathbf{F}}(x, y) =< M(x, y), N(x, y) >$, where $M(x, y)$ and $N(x, y)$ are continuous with continuous first partials in some open region D where $R \subset D$. Then

$$\oint_{\mathcal{C}} \overrightarrow{\mathbf{F}} \cdot d\overrightarrow{\mathbf{r}} = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) \, dA.$$

Recall: Why $\oint_{\mathcal{C}} \overrightarrow{\mathbf{F}} \cdot d\overrightarrow{\mathbf{r}} = \oint_{\mathcal{C}} M\,dx + \oint_{\mathcal{C}} N\,dy$:

If $\overrightarrow{\mathbf{F}}(x,y) =< M(x,y), N(x,y) >$, and if $\overrightarrow{\mathbf{r}}(t) =< x(t), y(t) >$, $a \le t \le b$ is parametrization for $\mathcal{C}$, then

$$\begin{aligned}
\oint_{\mathcal{C}} \overrightarrow{\mathbf{F}} \cdot d\overrightarrow{\mathbf{r}} &= \int_a^b \big\langle M\big(x(t), y(t)\big), N\big(x(t), y(t)\big)\big\rangle \cdot \big\langle x'(t), y'(t)\big\rangle\, dt \\
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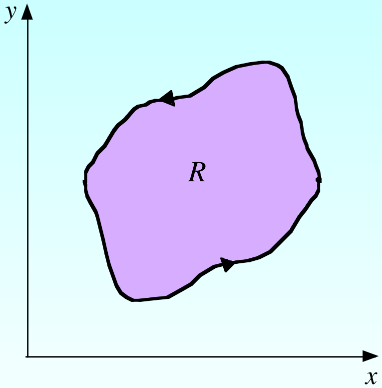
Special Case:

$\mathcal{C}$ (the boundary of R) is intersected by any vertical or horizontal line at most twice



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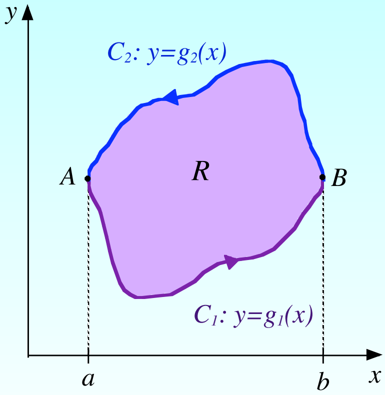

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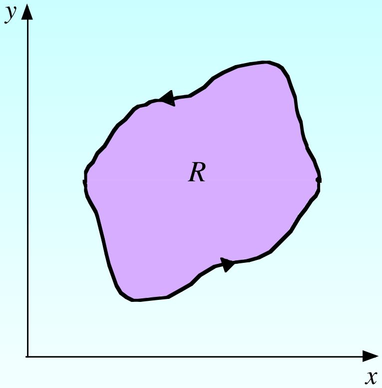

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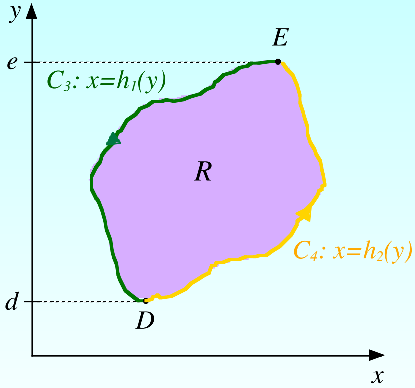

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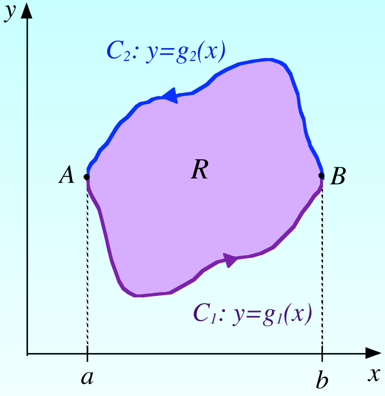

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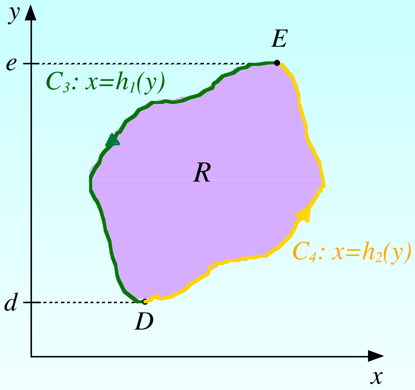

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  If you know the curvature at every point, you can construct the curve (give or take it's location in the plane)

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  - *Triple Integrals: 3D mass and center of mass, volume, a generalization of Green's Theorem, etc*

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  - *Stoke's Theorem- a* huge *generalization of the FTC.* Green's Theorem *is a special case, as is another important theorem, the* Divergence Theorem*.*