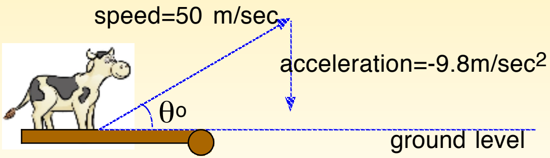
Solutions - In Class Work

A cow is launched from a catapult at ground level with an initial speed of 50 meters per second and at an angle of θ from the horizontal. Assume that the only force acting on the cow is gravity.

1. Find a vector-valued function $r(t)$ that describes the path travelled by the cow. Note that your answer will involve θ as a constant.

First, draw and label a picture!



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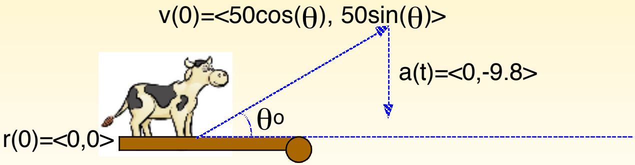
   Convert to vectors

   This is essentially a 2-dimensional problem. Translate into 2D vectors.



Find $\overrightarrow{\mathbf{r}}(t)$

Solutions - In Class Work

1.

$$\overrightarrow{\mathbf{a}}(t) =< 0, -9.8 > \qquad \overrightarrow{\mathbf{v}}(0) =< 50\cos(\theta), 50\sin(\theta) > \qquad \overrightarrow{\mathbf{r}}(0) =< 0, 0 > .$$

$$\begin{aligned}
\overrightarrow{\mathbf{v}}(t) &= \int \overrightarrow{\mathbf{a}}(t)\, dt =< 0, -9.8t > + \overrightarrow{\mathbf{c_1}} \\
\overrightarrow{\mathbf{v}}(0) =< 50\cos\theta, 50\sin\theta > &\implies \overrightarrow{\mathbf{c_1}} =< 50\cos\theta, 50\sin\theta > \\
\implies \overrightarrow{\mathbf{v}}(t) &= < 50\cos\theta, -9.8t + 50\sin\theta > \\
\implies \overrightarrow{\mathbf{r}}(t) &= \int \overrightarrow{\mathbf{v}}(t)\, dt \\
&= < 50\cos\theta t, -4.9t^2 + 50\sin\theta t > + \overrightarrow{\mathbf{c_2}} \\
\overrightarrow{\mathbf{r}}(0) =< 0, 0 > &\implies \overrightarrow{\mathbf{c_2}} =< 0, 0 > \\
\implies \overrightarrow{\mathbf{r}}(t) &= < 50\cos(\theta)t, -4.9t^2 + 50\sin(\theta)t >
\end{aligned}$$

Solutions - In Class Work

1. The path described by the cow:

$$\overrightarrow{\mathbf{r}}(t) = < 50\cos(\theta)t, -4.9t^2 + 50\sin(\theta)t > .$$

2. At what time will the cow hit the ground?

The cow will hit the ground the second time that the y-component of $\overrightarrow{\mathbf{r}}(t)$ is 0.

$$\begin{aligned} -4.9t^2 + 50\sin(\theta)t &= 0 \\ t(-4.9t + 50\sin(\theta)) &= 0 \\ t = 0 \quad \text{or} \quad t &= \frac{50}{4.9}\sin(\theta) \end{aligned}$$

Thus the cow will hit the ground after $\dfrac{50}{4.9}\sin(\theta)$ seconds.

Solutions - In Class Work

1. The path described by the cow:

$$\overrightarrow{\mathbf{r}}(t) = < 50\cos(\theta)t, -4.9t^2 + 50\sin(\theta)t > .$$

2. The cow will hit the ground after $t = \dfrac{50}{4.9}\sin(\theta)$ seconds.

3. How far from the launch point will the cow hit the ground?

How far from the launch point the cow will hit the ground is given by the x-component of the position function at the time the cow hits the ground.

$$50\cos(\theta)\frac{50}{4.9}\sin(\theta) = \frac{50^2}{4.9}\cos(\theta)\sin(\theta).$$

Thus the cow will land $\dfrac{50^2}{4.9}\cos(\theta)\sin(\theta)$ m from the launch point.

Solutions - In Class Work

The cow will land $\frac{50^2}{4.9}\cos(\theta)\sin(\theta)$ m from the launch point.

4. Find the value of θ that will maximize the horizontal distance traveled.

That is, maximize $d(\theta) = \dfrac{50^2}{4.9}\cos\theta\sin\theta$.

$$\begin{aligned} d'(\theta) &= \frac{50^2}{4.9}(\cos\theta\cos\theta - \sin\theta\sin\theta) \\ &= \frac{50^2}{4.9}(\cos^2\theta - \sin^2\theta) = \frac{50^2}{4.9}\cos(2\theta) \end{aligned}$$

$$d'(\theta) = 0 \implies \cos(2\theta) = 0 \implies 2\theta = \pi/2 \implies \theta = \pi/4$$

So assuming we're describing the angle as being between 0 and $\pi/2$, the maximum horizontal distance would be achieved if the cow-tapult is aimed at a 45° angle.