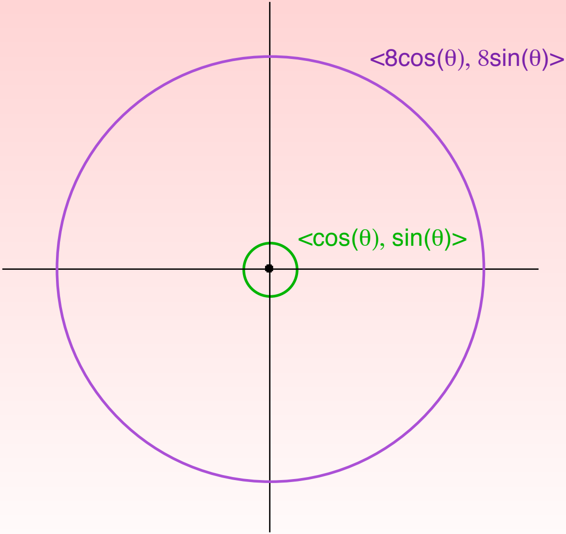
Intuition on Curvature:

A circle of small radius will curve more sharply – have greater curvature – than a circle of large radius:



Intuition on Curvature:

Curvature should be a characteristic of the geometric shape C traced out by a vector-valued function ... it shouldn't matter *how* the curve got traced out.

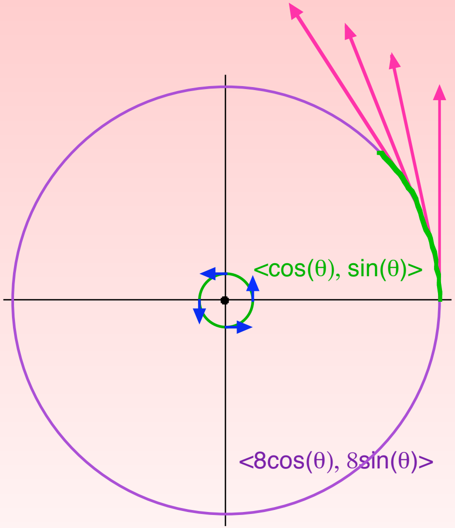
That is, the curvature of the unit circle shouldn't depend on whether we trace it out with

$\overrightarrow{\mathbf{r}}(t) =< \cos(t), \sin(t) >$ or $\overrightarrow{\mathbf{r}}(t) =< \sin(328t), -\cos(328t) >$.

Conclusion: Curvature should be independent of how the curve is parametrized.

On each of the circles, equally spaced tangent vectors over the same length of curve are shown.

On the smaller circle, which we intuitively describe as having greater curvature, the tangent vector changes direction much more over the same arclength than on the larger circle.



Conclusion: Curvature should be a measure of how the *direction* of the tangent vector changes with respect to arclength

Understanding $\left\|\frac{d\overrightarrow{\mathbf{T}}}{ds}\right\|$:

- **Recall:** when we parametrize wrt arclength, we choose a point R somewhere on our curve C. s is the signed distance along C of our point from R.
- The curve C is a curvy analogy to the x-axis, with the reference point R acting as the origin.
- $\frac{d\overrightarrow{\mathbf{T}}}{ds}$ is the rate the unit tangent vector changes as we move along the curve C from the reference point.
- Since $\|\overrightarrow{\mathbf{T}}\| = 1 =$a constant, only the direction of the unit tangent vector is changing, so $\left\|\frac{d\overrightarrow{\mathbf{T}}}{ds}\right\|$ measures the rate the direction of the tangent vector changes as we move along C.
- Since the tangent vector points in the direction of the curve, this is a measure of the curviness

Recall:

Since

$$s(t) = \int_{t_0}^{t} \|\overrightarrow{\mathbf{r}}\,'(u)\|\, du,$$

by the Fundamental Theorem of Calculus,

$$\frac{ds}{dt} = \|\overrightarrow{\mathbf{r}}\,'(t)\|.$$

In Class Work

Compute the curvature of the curve traced out by $\overrightarrow{\mathbf{r}}(t) =< t^2, 3t, t^3 >$ when $t = 1$.