Consider the points $P_1 = (-1, 3, 4)$ and $P_2 = (2, -1, 3)$.
1. Find an equation of the line through P_1 and P_2.

For a line, we need a point and a direction.

For the point, can choose P_1 or P_2.
I'll use $P_1 = (-1, 3, 4)$.

For the direction, can use $\overrightarrow{\mathbf{P_1P_2}}$ or $\overrightarrow{\mathbf{P_2P_1}}$.
I'll use $\overrightarrow{\mathbf{P_1P_2}} = < 3, -4, -1 >$.

A point (x, y, z) is on the line iff the vector $< x + 1, y - 3, z - 4 >$ is parallel to $< 3, -4, -1 >$, or in other words iff

$$< x + 1, y - 3, z - 4 > = t < 3, -4, -1 >$$

$$\boxed{\Longrightarrow x = -1 + 3t, y = 3 - 4t, z = 4 - t.}$$

Consider the points $Q_1 = (2, 0, 3)$, $Q_2 = (4, 1, -2)$, and $Q_3 = (5, 1, 6)$.
2. Find an equation of the plane that contains Q_1, Q_2, and Q_3.

For a plane, we need a point and a normal (orthogonal) vector.

For the point, can choose Q_1, Q_2, or Q_3.
I'll use Q_1.

Find a normal vect by taking $\overrightarrow{\mathbf{Q_1Q_2}} \times \overrightarrow{\mathbf{Q_1Q_3}}$ or any other combo.
I'll use $\overrightarrow{\mathbf{Q_1Q_2}} \times \overrightarrow{\mathbf{Q_1Q_3}}$.

$$\begin{aligned} \overrightarrow{\mathbf{Q_1Q_2}} &= \ < 2, 1, -5 >, \qquad \overrightarrow{\mathbf{Q_1Q_3}} =< 3, 1, 3 > \\ \Longrightarrow \overrightarrow{\mathbf{Q_1Q_2}} \times \overrightarrow{\mathbf{Q_1Q_3}} &= \ < 3+5, -(6+15), 2-3 >=< 8, -21, -1 > \end{aligned}$$

A point $P(x, y, z)$ is on the plane iff $\overrightarrow{\mathbf{PQ_1}} \cdot < 8, -21, -1 >= 0$, i.e. iff
$< x-2, y, z-3 > \cdot < 8, -21, -1 >= 0 \Rightarrow 8x - 16 - 21y - z + 3 = 0$

$$\boxed{\Longrightarrow 8x - 21y - z = 13.}$$

Consider the points

$P_1 = (-1, 3, 4) \quad P_2 = (2, -1, 3)$

$Q_1 = (2, 0, 3) \qquad Q_2 = (4, 1, -2) \quad Q_3 = (5, 1, 6)$

1. Find an equation of the line through P_1 and P_2.

   Found: $x = -1 + 3t, y = 3 - 4t, z = 4 - t$

2. Find an equation of the plane that contains Q_1, Q_2, and Q_3.

   Found: $8x - 21y - z = 13$.

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3. Find the point(s) where the line in #1 intersects the plane in #2.

4. Find the line of intersection of the plane in #2 with the plane $5x + y + 11z = 42$.