Notation and Vocabulary for $\mathbb{R}^n$:

- One dimension = real-number line = $\mathbb{R}$.
- 2-dimensional space = plane $=\mathbb{R}^2$. We'll call it "2-space" sometimes.
- 3-dimensional space= space$=\mathbb{R}^3$. We'll call it "3-space".
- In general, n dimensions$=\mathbb{R}^n=$ n-space.

3-dimensional space is the highest number of dimensions we can readily graph.

Most ideas related to points in 2-space have a natural extension to 3-, or n-, space.

- Distance in 3-space

$$\text{distance} = d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$

- Just as a circle of radius r centered at (a, b) consists of all points (x, y) that are a distance r away from (a, b), so

$$(x - a)^2 + (y - b)^2 = r^2,$$

  a sphere of radius r centered at (a, b, c) is of the form

$$(x - a)^2 + (y - b)^2 + (z - c)^2 = r^2.$$

Vector Basics

- **scalar**=quantity described by *magnitude* only.

  Examples: area, length, mass, temperature

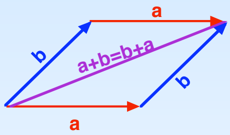
- **vector**=quantity described by *magnitude* and *direction*.

  Examples: wind velocity, force, displacement

- Represent vectors with arrows
  - direction of arrow= direction of vector
  - length of the arrow = magnitude of the vector

Vector Basics

- Geometry of Vector Addition:



- **zero vector**, $\overrightarrow{\mathbf{0}} =$ vector whose initial and terminal points are the same – a vector of length 0.

- $\overrightarrow{\mathbf{0}} + \overrightarrow{\mathbf{v}} = \overrightarrow{\mathbf{v}} = \overrightarrow{\mathbf{v}} + \overrightarrow{\mathbf{0}}$

- **scalar multiple** $k\overrightarrow{\mathbf{v}}$ = vector of length $|k| \times$ length of $\overrightarrow{\mathbf{v}}$, direction is *the same* as that of $\overrightarrow{\mathbf{v}}$ if $k > 0$ and the opposite of that of $\overrightarrow{\mathbf{v}}$ if $k < 0$.

- $(-1)\overrightarrow{\mathbf{v}}$=**the opposite** of $\overrightarrow{\mathbf{v}}$; can write as $-\overrightarrow{\mathbf{v}}$.

Vector Basics

- Given any vector $\overrightarrow{\mathbf{v}}$, if you position it so that it's initial point is at the origin and it's terminal point is at the point $P(v_1, v_2)$, or in 3-space $P(v_1, v_2, v_3)$, then we call these these coordinates the components of the vector $\overrightarrow{\mathbf{v}}$, and write

$$\overrightarrow{\mathbf{v}} = < v_1, v_2 > \qquad \text{or} \qquad \overrightarrow{\mathbf{v}} = < v_1, v_2, v_3 > .$$

- If we have a vector with initial point P_1 and terminal point P_2, don't have to re-draw it with its initial point at the origin:

$$\begin{aligned} \overrightarrow{\mathbf{P_1 P_2}} &= < x_2 - x_1, y_2 - y_1 > \\ \overrightarrow{\mathbf{P_1 P_2}} &= < x_2 - x_1, y_2 - y_1, z_2 - z_1 > . \end{aligned}$$

Vector Basics

- Addition and subtraction are done component-wise:

Example: If $\overrightarrow{\mathbf{a}} =< 3, 0, >$ and $\overrightarrow{\mathbf{b}} =< 2, 2 >$,

$$\begin{aligned} \overrightarrow{\mathbf{a}} + \overrightarrow{\mathbf{b}} &= < 3, 0 > + < 2, 2 > = < 5, 2 > \\ \overrightarrow{\mathbf{a}} - \overrightarrow{\mathbf{b}} &= < 3, 0 > - < 2, 2 > = < 1, -2 > . \end{aligned}$$

When we write a vector using the notation $\overrightarrow{\mathbf{v}} = < x, y, z >$, we know that if we choose to draw this vector so that it's initial point is at the origin, it's terminal point is at the point (x, y, z).

$$\text{Length of vector} = \|\overrightarrow{\mathbf{v}}\| = \sqrt{x^2 + y^2 + z^2}.$$

This length is also called *magnitude* and *norm*.

A **unit vector** is a vector of length 1.

- In 2-space, $< 1, 0 >$ and $< 0, 1 >$ are unit vectors pointing in the direction of the positive x and y axes.

  These are commonly written as

  - $\overrightarrow{\mathbf{i}} =< 1, 0 >$
  - $\overrightarrow{\mathbf{j}} =< 0, 1 >$

  Any vector $< x, y >$ can also be written $x\overrightarrow{\mathbf{i}} + y\overrightarrow{\mathbf{j}}$.

- In 3-space, we expand on this:

  - $\overrightarrow{\mathbf{i}} =< 1, 0, 0 >$
  - $\overrightarrow{\mathbf{j}} =< 0, 1, 0 >$
  - $\overrightarrow{\mathbf{k}} =< 0, 0, 1 >$

  Any vector $< x, y, z >$ can also be written $x\overrightarrow{\mathbf{i}} + y\overrightarrow{\mathbf{j}} + z\overrightarrow{\mathbf{k}}$.

Dot Product

- If $\overrightarrow{\mathbf{v}} =< v_1, v_2 >$ and $\overrightarrow{\mathbf{w}} =< w_1, w_2 >$, then

$$\overrightarrow{\mathbf{v}} \cdot \overrightarrow{\mathbf{w}} =< v_1, v_2 > \cdot < w_1, w_2 > \stackrel{\text{def}}{=} v_1 w_1 + v_2 w_2$$

- If $\overrightarrow{\mathbf{v}} =< v_1, v_2, v_3 >$ and $\overrightarrow{\mathbf{w}} =< w_1, w_2, w_3 >$, then

$$\overrightarrow{\mathbf{v}} \cdot \overrightarrow{\mathbf{v}} =< v_1, v_2, v_3 > \cdot < w_1, w_2, w_3 > \stackrel{\text{def}}{=} v_1 w_1 + v_2 w_2 + v_3 w_3.$$

- In general, if $\overrightarrow{\mathbf{v}} =< v_1, v_2, \ldots, v_n >$ and $\overrightarrow{\mathbf{w}} =< w_1, w_2, \ldots, w_n >$, then

$$\overrightarrow{\mathbf{v}} \cdot \overrightarrow{\mathbf{w}} =< v_1, \ldots, v_n > \cdot < w_1, \ldots, w_n > \stackrel{\text{def}}{=} v_1 w_1 + v_2 w_2 + \cdots + v_n w_n.$$

Dot Product

$$\|\overrightarrow{\mathbf{v}}\|^2 = \left(\sqrt{v_1^2 + \cdots + v_n^2}\right)^2 = v_1^2 + v_2^2 + \cdots + v_n^2 = \overrightarrow{\mathbf{v}} \cdot \overrightarrow{\mathbf{v}}.$$

Let $\overrightarrow{\mathbf{a}} =< 3, 5 >$, $\overrightarrow{\mathbf{b}} =< 7, 2 >$, and $\overrightarrow{\mathbf{c}} =< 1, 3, -2 >$.

1. Draw $\overrightarrow{\mathbf{a}}$ and $\overrightarrow{\mathbf{b}}$ on the same set of axes. On a separate set of axes, draw $\overrightarrow{\mathbf{c}}$.
2. Draw $\overrightarrow{\mathbf{a}} + \overrightarrow{\mathbf{b}}$ and $\overrightarrow{\mathbf{a}} - \overrightarrow{\mathbf{b}}$.
3. Find $\overrightarrow{\mathbf{a}} \cdot \overrightarrow{\mathbf{b}}$, if possible. If it's not possible, why not?
4. Find $\overrightarrow{\mathbf{b}} \cdot \overrightarrow{\mathbf{c}}$, if possible. If it's not possible, why not?
5. Is the angle between $\overrightarrow{\mathbf{a}}$ and $\overrightarrow{\mathbf{b}}$ acute, orthogonal, or obtuse?
   Do (5) without calculating the angle.
6. Find the angle between $\overrightarrow{\mathbf{a}}$ and $\overrightarrow{\mathbf{b}}$.
7. Find two vectors perpendicular to $\overrightarrow{\mathbf{b}}$.
8. Find three vectors perpendicular to $\overrightarrow{\mathbf{c}}$.