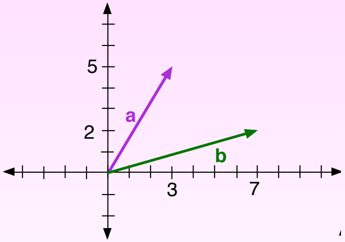
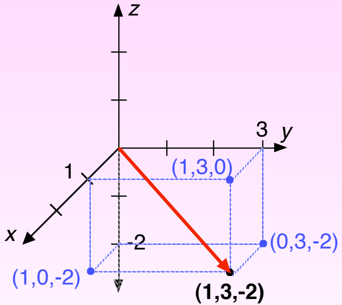
Let $\overrightarrow{\mathbf{a}} =< 3, 5 >$, $\overrightarrow{\mathbf{b}} =< 7, 2 >$, and $\overrightarrow{\mathbf{c}} =< 1, 3, -2 >$.

1. Draw $\overrightarrow{\mathbf{a}}$ and $\overrightarrow{\mathbf{b}}$ on the same set of axes. On a separate set of axes, draw $\overrightarrow{\mathbf{c}}$.



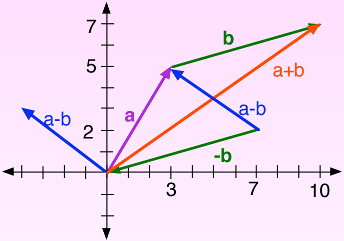
$\overrightarrow{\mathbf{a}}$, $\overrightarrow{\mathbf{b}}$



$\overrightarrow{\mathbf{c}}$

Let $\overrightarrow{\mathbf{a}} =< 3, 5 >$, $\overrightarrow{\mathbf{b}} =< 7, 2 >$, and $\overrightarrow{\mathbf{c}} =< 1, 3, -2 >$.

2. Draw $\overrightarrow{\mathbf{a}} + \overrightarrow{\mathbf{b}}$ and $\overrightarrow{\mathbf{a}} - \overrightarrow{\mathbf{b}}$.



Let $\overrightarrow{\mathbf{a}} =< 3, 5 >$, $\overrightarrow{\mathbf{b}} =< 7, 2 >$, and $\overrightarrow{\mathbf{c}} =< 1, 3, -2 >$.

3. Find $\overrightarrow{\mathbf{a}} \cdot \overrightarrow{\mathbf{b}}$, if possible. If it's not possible, why not?

$$\overrightarrow{\mathbf{a}} \cdot \overrightarrow{\mathbf{b}} =< 3, 5 > \cdot < 7, 2 >= 21 + 10 = 31.$$

4. Find $\overrightarrow{\mathbf{b}} \cdot \overrightarrow{\mathbf{c}}$, if possible. If it's not possible, why not?

Because $\overrightarrow{\mathbf{b}}$ sits in 2-dimensions and $\overrightarrow{\mathbf{c}}$ sits in 3, this is not possible.

Let $\overrightarrow{\mathbf{a}} =< 3, 5 >$, $\overrightarrow{\mathbf{b}} =< 7, 2 >$, and $\overrightarrow{\mathbf{c}} =< 1, 3, -2 >$.

5. Is the angle between $\overrightarrow{\mathbf{a}}$ and $\overrightarrow{\mathbf{b}}$ acute, orthogonal, or obtuse?
Do (5) without calculating the angle.

We know that

$$\overrightarrow{\mathbf{a}} \cdot \overrightarrow{\mathbf{b}} = \|a\| \, \|b\| \cos(\theta).$$

Since the magnitudes $\|\overrightarrow{\mathbf{a}}\|$, $\|\overrightarrow{\mathbf{b}}\|$ are positive, the sign of $\overrightarrow{\mathbf{a}} \cdot \overrightarrow{\mathbf{b}}$ depends completely on the sign of $\cos(\theta)$ ... which in turn depends completely on whether the angle is acute or obtuse.

Thus since $\overrightarrow{\mathbf{a}} \cdot \overrightarrow{\mathbf{b}} > 0$, θ must be acute.

Let $\overrightarrow{\mathbf{a}} =< 3, 5 >$, $\overrightarrow{\mathbf{b}} =< 7, 2 >$, and $\overrightarrow{\mathbf{c}} =< 1, 3, -2 >$.

6. Find the angle between $\overrightarrow{\mathbf{a}}$ and $\overrightarrow{\mathbf{b}}$.

$$\begin{aligned}
\overrightarrow{\mathbf{a}} \cdot \overrightarrow{\mathbf{b}} &= \|a\| \, \|b\| \cos(\theta) \\
31 &= \sqrt{9+25}\sqrt{49+4}\cos(\theta) \\
\frac{31}{\sqrt{34}\sqrt{53}} &= \cos(\theta) \\
\theta &= \arccos\left(\frac{31}{\sqrt{1802}}\right) \approx \arccos(0.73027) \\
\theta &= .752 \text{ radians}
\end{aligned}$$

Let $\overrightarrow{\mathbf{a}} =< 3, 5 >$, $\overrightarrow{\mathbf{b}} =< 7, 2 >$, and $\overrightarrow{\mathbf{c}} =< 1, 3, -2 >$.

7. Find two vectors perpendicular to $\overrightarrow{\mathbf{b}}$.

In order to be perpendicular to $\overrightarrow{\mathbf{b}}$, the dot product must be zero.

$$< x, y > \cdot < 7, 2 >= 0 \Rightarrow 7x + 2y = 0.$$

The vectors $< -2, 7 >$ and $< 2, -7 >$ seem like the most obvious choices.

Any other choice will turn out to be a multiple of these two, since there are only two directions perpendicular to this vector in 2-space.

Let $\overrightarrow{\mathbf{a}} =< 3, 5 >$, $\overrightarrow{\mathbf{b}} =< 7, 2 >$, and $\overrightarrow{\mathbf{c}} =< 1, 3, -2 >$.

8. Find three vectors perpendicular to $\overrightarrow{\mathbf{c}}$.

$$< x, y, z > \cdot < 1, 3, -2 >= 0 \Rightarrow 1x + 3y - 2z = 0.$$

- If $x = 1$ and $y = 1$, then $1 + 3 - 2z = 0$, so $z = 2$. Thus $< 1, 1, 2 >$ is orthogonal to $\overrightarrow{\mathbf{c}}$
- If $x = 1$ and $z = 1$, then $1 + 3y - 2 = 0$, so $y = \frac{1}{3}$. Thus $< 1, \frac{1}{3}, 1 >$ is orthogonal to $\overrightarrow{\mathbf{c}}$ as well.
- If $y = 1$ and $z = 1$, then $x + 3 - 2 = 0$, so $x = -1$. Thus $< -1, 1, 1 >$ is a third vector orthogonal to $\overrightarrow{\mathbf{c}}$.

Notice: none of these three are multiples of any of the others. In space, there are an infinite number of directions perpendicular to any given vector.