Solutions - In Class Work

1.(a) Express $x = u \qquad y = 3u^2 + 2v^2 \qquad z = v$ using relationship(s) between x, y, and z.

For this, we simply notice that y is already written in terms of both z and z:

$$y = 3x^2 + 2z^2.$$

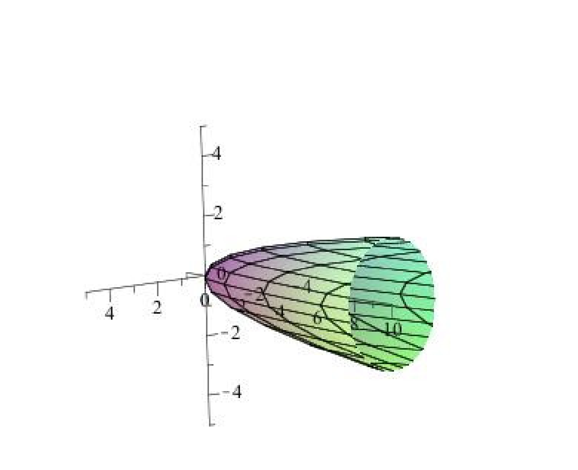
1. (b) Sketch traces & identify the surface without using Maple or the book.

- **Trace in the xy-plane:** $y = 3x^2 \Longrightarrow$ parabola.
- **Trace in the xz-plane:** $3x^2 + 2z^2 = 0 \Longrightarrow$ point $(0, 0, 0)$.
- **Traces parallel to the xz-plane:** When $y = k > 0$, $3x^2 + 2z^2 = k \Longrightarrow$ ellipses.
- **Trace in the yz-plane:** $y = 2z^2 \Longrightarrow$ parabola

This surface lies on an **Elliptic paraboloid**

1. (c) Plot on Maple to check. Compare how Maple plots the parametric equations with how the surface looks using implicitplot3d.

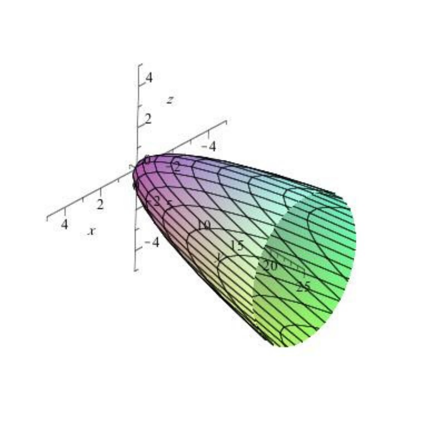
- `[u, 3*u^2+2*v^2, v]];`
  right click on blue result
  `plots-plot builder - 3D parametric plot`



1. (c) Plot on Maple to check. Compare how Maple plots the parametric equations with how the surface looks using implicitplot3d.

- `y=3*x^2+2*z^2;`
  right click on blue result

  `plots-3D implicit plot - x,y,z`



2. (a) Express $x = v \sinh(u) \qquad y = 4v^2 \qquad z = v \cosh(u)$ using relationship(s) between x, y, and z.

$$
\begin{aligned}
z^2 - x^2 &= v^2(\cosh^2(u) - \sinh^2(u)) \\
&= v^2 \\[1em]
4z^2 - 4x^2 &= y
\end{aligned}
$$

Be Careful! Notice that in the original parametric description of the surface, y will always be non-negative. **However**, in this new relationship we've found, y can be negative as well as 0 or positive.
Thus our parametric surface will not be the entirety of the surface we graph from the relationship $y = 4z^2 - 4x^2$.

2 (b) Sketch traces & identify the surface without using Maple or the book.

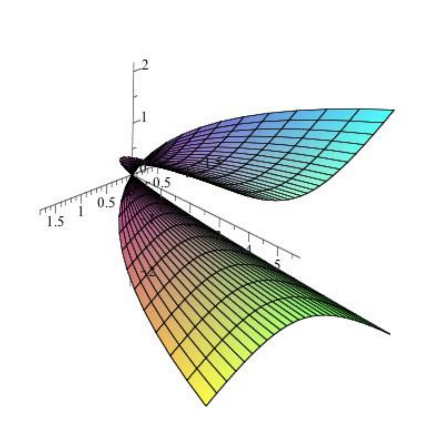
- **Trace in the** xy**-plane:** $y = -4x^2 \implies$ leftward opening horizontal parabola.
  This trace is not part of our original surface, since $y < 0$.
- **Trace in the** xz**-plane:** $4z^2 - 4x^2 = 0 \implies z = \pm x \implies$ two lines
- **Trace parallel to the** xz**-plane:** $y = k \implies 4z^2 - 4x^2 = k \implies$ hyperbolas
- **Trace in the** yz**-plane:** $y = 4z^2 \implies$ rightward opening vertical parabola

Based on the cross-sections being parabolas in 2 directions and hyperbolas in one direction, it sounds like our surface lies on a **hyperbolic paraboloid**.

2. (c) Plot on Maple to check. Compare how Maple plots the parametric equations with how the surface looks using implicitplot3d.

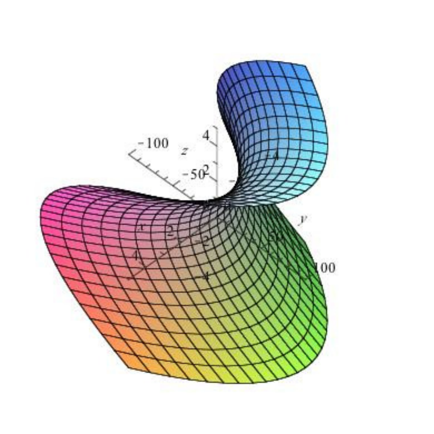
- `[v*sinh(u), 4*v^2, v*cosh(u)];`
  right click on blue result

  `plots-plot builder - 3D parametric plot`



2. (c) Plot on Maple to check. Compare how Maple plots the parametric equations with how the surface looks using implicitplot3d.

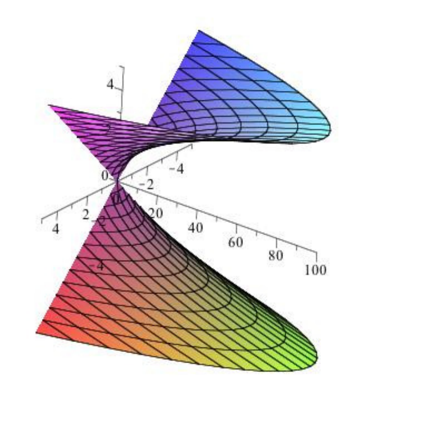
- `y=4*z^2-4*x^2;`
  right click on blue result
  `plots-3D implicit plot - x,y,z`



Notice that there's a lot more to this surface than in the previous graph– remember, that's because our parametrization was for just positive y, but this surface has both positive and negative y. I restricted to positive y on the next slide.

2. (c) Plot on Maple to check. Compare how Maple plots the parametric equations with how the surface looks using implicitplot3d.

- `y=4*z^2-4*x^2;`
  right click on blue result
  `plots-3D implicit plot - x,y,z`



This is the surface

$$y = 4z^2 - 4x^2, y \geq 0$$

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