Let $f(x,y) = \dfrac{6(x-1)^3(y-2)}{2(x-1)^4 + (y-2)^4}$. Consider $\lim\limits_{(x,y)\to(1,2)} f(x,y)$.
Remember: all paths we try must go through the point $(1,2)$!

1. Find the limit as you approach the origin along the line $x = 1$.

$$\lim_{(1,y)\to(1,2)} f(x,y) = \lim_{y\to 2} \frac{0}{(y-2)^4} = 0.$$

2. Find the limit as you approach the origin along the line $y = 2$.

$$\lim_{(x,2)\to(1,2)} f(x,y) = \lim_{x\to 1} \frac{0}{2(x-1)^4} = 0.$$

Let $f(x,y) = \dfrac{6(x-1)^3(y-2)}{2(x-1)^4 + (y-2)^4}$. Consider $\displaystyle\lim_{(x,y)\to(1,2)} f(x,y)$.
Remember: all paths we try must go through the point $(1,2)$!

3. Find the limit as you approach the origin along the line $y = x + 1$. (*Notice that the point* $(1,2)$ *lies on this line.*)

$$\begin{aligned}
\lim_{(x,x+1)\to(1,2)} f(x,y) &= \lim_{x\to 1} \frac{6(x-1)^3(x-1)}{2(x-1)^4 + (x-1)^4} \\
&= \lim_{x\to 1} \frac{6(x-1)^4}{3(x-1)^4} \\
&= 2.
\end{aligned}$$

Let $f(x,y) = \dfrac{6(x-1)^3(y-2)}{2(x-1)^4 + (y-2)^4}$. Consider $\displaystyle\lim_{(x,y)\to(1,2)} f(x,y)$.
Remember: all paths we try must go through the point $(1,2)$!

4. Find the limit as you approach the origin along the line $y = m(x-1) + 2$.
   (*Again, notice that the point* $(1,2)$ *lies on this line.*)

$$
\begin{aligned}
\lim_{(x,m(x-1)+2)\to(1,2)} f(x,y) &= \lim_{x\to 1} \frac{6(x-1)^3\big(m(x-1)\big)}{2(x-1)^4 + \big(m(x-1)\big)^4} \\
&= \lim_{x\to 1} \frac{6m(x-1)^4}{(2+m^4)(x-1)^4} \\
&= \frac{6m}{2+m^4} \lim_{x\to 1} \frac{(x-1)^4}{(x-1)^4} \\
&= \frac{6}{2+m^4}.
\end{aligned}
$$

Let $f(x,y) = \dfrac{6(x-1)^3(y-2)}{2(x-1)^4+(y-2)^4}$. Consider $\displaystyle\lim_{(x,y)\to(1,2)} f(x,y)$.

5. What can you conclude about $\displaystyle\lim_{(x,y)\to(1,2)} f(x,y)$?

   If we approach $(1,2)$ along some line through $(1,2)$, what z-value we approach depends upon which line we're traveling along.

   Since we approach different values along different paths, the limit does not exist.

   (We could have concluded this after #3, but I wanted to demonstrate that the different limit along $y = x+1$ wasn't a fluke; instead, getting the *same* limit along $x = 1$ and $y = 2$ was the fluke.)