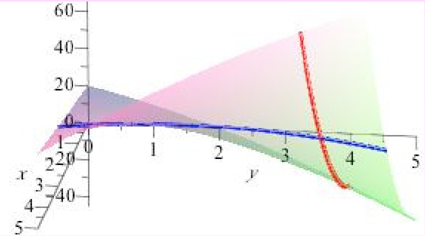
1. Let $g(x, y) = x^2y + 4x + y^2 - 8y + xy + 20$. Find $g_x(3, 4)$ and $g_y(3, 4)$. Look at the graph below and decide whether your results agree with what you're seeing.



$$\begin{aligned} g_x(x, y) &= \frac{\partial}{\partial x} g(x, y) = 2xy + 4 + y \\ g_x(3, 4) &= 2(3)(4) + 4 + 4 = 32 \end{aligned}$$

$$\begin{aligned} g_y(x, y) &= x^2 - 2y - 8 + x \\ g_y(3, 4) &= 3^2 - 2(4) - 8 + 3 = -4 \end{aligned}$$

- $g_x(3, 4)$ = slope of the curve $\langle x, 4, g(x, 4)\rangle$ at the point $(3, 4, g(3, 4))$. This curve is increasing rapidly there, which agrees with $g_x(3, 4) = 32$.
- $g_y(3, 4)$ = slope of the curve $\langle 3, y, g(3, y)\rangle$ at the point $(3, 4, g(3, 4))$. This curve is decreasing comparatively slowly there, which corresponds with $g_y(3, 4) = -4$.

2. For $g(x, y) = x^2y + 4x + y^2 - 8y + xy + 20$, find all points where $g_x(x, y)$ and $g_y(x, y)$ are both zero.

Notice that if I just set the two partials equal to each other from the beginning, I'm trying to find **every** *point where they two partials are equal, and not using that they're in fact both 0.*

$$
\begin{aligned}
& g_x(x, y) = 0 \\
\Longrightarrow\ & 2xy + 4 + y = 0 \\
\Longrightarrow\ & (2x + 1)y = -4 \\
\Longrightarrow\ & y = -\frac{4}{2x + 1}
\end{aligned}
$$

$$
\begin{aligned}
& g_y(x, y) = 0 \\
\Longrightarrow\ & x^2 - 2y - 8 + x = 0 \\
\Longrightarrow\ & 2y = x^2 + x - 8
\end{aligned}
$$

2. (continued) For $g(x, y) = x^2y + 4x + y^2 - 8y + xy + 20$, find all points where $g_x(x, y)$ and $g_y(x, y)$ are both zero.

$$g_x(x,y) = 0 \Longrightarrow y = -\frac{4}{2x+1} \qquad g_y(x,y) = 0 \Longrightarrow 2y = x^2 + x - 8.$$

$$\begin{aligned}
g_x(x,y) = 0 = g_y(x,y) \quad &\Longrightarrow \quad x^2 + x - 8 = 2y = 2\left(-\frac{4}{2x+1}\right) \\
&\Longrightarrow \quad 2x^3 + x^2 + 2x^2 + x - 16x - 8 = -8 \\
&\Longrightarrow \quad 2x^3 + 3x^2 - 15x = 0 \\
&\Longrightarrow \quad x(2x^2 + 3x - 15) = 0 \\
&\Longrightarrow \quad x = 0 \text{ or } x = \frac{-3 \pm \sqrt{9 - 120}}{4} \\
&\Longrightarrow \quad x = 0 \\
&\Longrightarrow \quad y = -4
\end{aligned}$$

Thus the only point where $g_x(x, y) = 0 = g_y(x, y)$ is the point $(0, -4)$.

2. (continued) **What is the significance of the point** $(0, -4)$ **on the graph of** $g(x, y) = x^2y + 4x + y^2 - 8y + xy + 20$**?**

2. (continued) **What is the significance of the point** $(0, -4)$ **on the graph of** $g(x, y) = x^2y + 4x + y^2 - 8y + xy + 20$**?**

- **If** a "nice" function $f(x, y)$ has a maximum or minimum value at (a, b), **then** we must have $f_x(a, b) = 0 = f_y(a, b)$.
- Thus for "nice" functions, local max's and min's **can only occur** at places where both $f_x(a, b) = 0 = f_y(a, b)$, so we have *critical points* – points that are *possible* min's and max's.

In this case, the point $(0, -4)$ is **a candidate** to be a max or a min, although without further investigation, we can't know for sure.

3. For $g(x,y) = x^2y + 4x + y^2 - 8y + xy + 20$, find g_{xy} and g_{yx}.

$$\begin{aligned} g_{xy} &= (g_x)_y \\ &= (2xy - 4 + y)_y \\ &= 2x + 1 \end{aligned}$$

$$\begin{aligned} g_{yx} &= (g_y)_x \\ &= (x^2 + 2y - 8 + x)_x \\ &= 2x + 1 \end{aligned}$$

Question: For this one example, we got that the mixed partials are the same. Coincidence?