Recall:

Consider a function $f(x, y)$.

If we hold y constant and look at the rate f changes as we move in the x -direction, we have reduced to a function of just x, which we know how to differentiate.

We call such a derivative the **partial derivative of f with respect to x**, denoted $f_x(x, y)$ or $\frac{\partial f}{\partial x}(x, y)$.

Similarly, if we hold x constant, we have reduced to a function of just y, which we can differentiate to find the **partial derivative of f with respect to y**, and denote $f_y(x, y)$ or $\frac{\partial f}{\partial y}(x, y)$.

Recall:

On Friday, you worked through the following problem:

1. Let $g(x, y) = x^2y + 4x - y^2 - 8y + xy + 20$. Find all points where $g_x(x, y)$ and $g_y(x, y)$ are both zero. What is the significance of these points?

We found that the only such point is the point $(0, -4)$.

But what is the significance of this point?

1. Find and classify (as best you can) all critical points of the function $h(x, y) = x^2 - 4x - 23 - y^3 - 9y^2 - 27y + xy$

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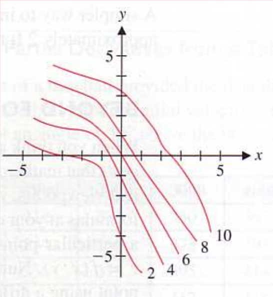
2. Use the contour plot at right to estimate $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ at the origin.



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3. The **wave equation** is the partial differential equation $c^2 \dfrac{\partial^2 f}{\partial x^2} = \dfrac{\partial^2 f}{\partial t^2}$. Show that the functions $f_n(x, t) = \sin(n\pi x) \cos(n\pi ct)$ satisfy the wave equation, for any positive integer n and any constant c.

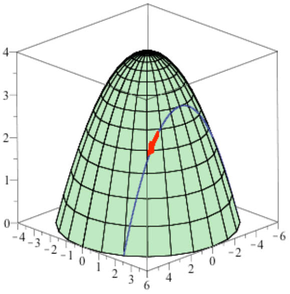
Preview of §12.4:

- If $f(x)$ is a differentiable function, we can use the tangent line at $x = x_0$,

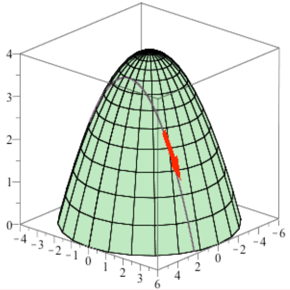
$$y = L(x) = f'(x_0)(x - x_0) + f(x_0)$$

  as a **linear approximation** of f at x_0.

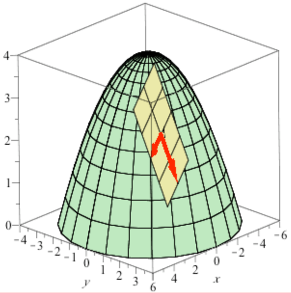
- In the same way, for a function $f(x, y)$, the *tangent plane* at (a, b) will give a linear approximation of f at (a, b).



Above, the red arrow represents the line to the curve formed by the intersection of the plane $y = b$ with the surface $z = f(x, y)$.



This arrow represents the tangent line to the curve at (a, b) when $x = a$ is fixed.



These two lines (and every other tangent line to a curve on the surface that goes through the point $(a, b, f(a, b))$ will lie on our **tangent plane**, shown in yellow.

Finding the tangent plane:

In order to find the equation of a plane, we need: **a point** and **a normal vector**.

- **Point:** use the point of tangency, $(a, b, f(a, b))$.
- **Normal vector:** Find it by first finding the direction vectors for each tangent line, then taking the cross product.