1. Find and classify (as best you can) all critical points of the function $h(x, y) = x^2 - 4x - 23 - y^3 - 9y^2 - 27y + xy$

$$h_x = 2x - 4 + y \qquad h_y = -3y^2 - 18y - 27 + x$$

$$h_x = 0 \Rightarrow x = 2 - y/2 \qquad h_y = 0 \Rightarrow -3y^2 - 18y - 27 + (2 - y/2) = 0$$

$$\begin{aligned}
\Rightarrow -3y^2 - 37y/2 - 25 &= 0 \\
\Rightarrow 6y^2 + 37y + 50 &= 0 \\
\Rightarrow y &= \frac{-37 \pm \sqrt{(37)^2 - 4(6)(50)}}{2(6)} \\
\Rightarrow y &= \frac{-37 \pm \sqrt{1369 - 1200}}{12} \\
\Rightarrow y &= \frac{-37 \pm 13}{12} \\
\Rightarrow y &= -2 \text{ or } y = -\frac{25}{6}
\end{aligned}$$

1. *(continued)* Find and classify (as best you can) all critical points of the function $h(x, y) = x^2 - 4x - 23 - y^3 - 9y^2 - 27y + xy$

We found that $h_x = 2x - 4 + y$ and $h_y = -3y^2 - 18y - 27 + x$.

When we solved $h_x = 0$ and $h_y = 0$, we found

$$y = -2 \qquad \text{or} \qquad y = -\frac{25}{6}.$$

Using $h_x = 0 \Rightarrow x = 2 - y/2$, the two critical points are $(3, -2)$ and $(\frac{49}{12}, -\frac{25}{6})$.

1. *(continued)* Find and classify (as best you can) all critical points of the function $h(x, y) = x^2 - 4x - 23 - y^3 - 9y^2 - 27y + xy$

What can we figure out from the 2nd partials?

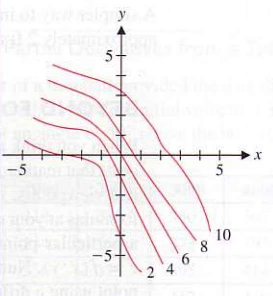
$$h_{xx} = 2 \qquad h_{yy} = -6y - 18.$$

At $(3, -2)$, $h_{xx}(3, -2) = 2 > 0 \Rightarrow h$ is concave up parallel to x; $h_{yy}(3, -2) = 0$ is uninformative.

At $(49/12, -25/6)$,$h_{xx}(49/12, -25/6) = 2 > 0 \Rightarrow h$ is concave up parallel to x; $h_{yy}(49/12, -25/6) = 7 > 0 \Rightarrow h$ is concave up parallel to y.

Thus we know that neither point is a local maximum (since in at least one direction, the surface is concave up in each case). They each could be minima or saddle points. Look at contour plot to decide.

2. Use the contour plot at right to estimate $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ at the origin.



This is a homework problem.

The key idea here is that $\frac{\partial f}{\partial x}(0,0) \approx \left.\frac{\Delta f}{\Delta x}\right|_{(0,0)}$, and that we know the value of f along the contour plots.

3. The **wave equation** is the partial differential equation $c^2 \dfrac{\partial^2 f}{\partial x^2} = \dfrac{\partial^2 f}{\partial t^2}$. Show that the functions $f_n(x,t) = \sin(n\pi x)\cos(n\pi ct)$ satisfy the wave equation, for any positive integer n and any constant c.

This is also a homework problem.

In this problem, the key idea is finding $\dfrac{\partial^2 f}{\partial x^2}$ and $\dfrac{\partial^2 f}{\partial t^2}$, and then showing that multiplying $\dfrac{\partial^2 f}{\partial x^2}$ by c^2 will produce $\dfrac{\partial^2 f}{\partial t^2}$.