Let $f(x,y) = e^{-x^2-y^2}$.

1. Find the equation of the plane tangent to $z = f(x,y)$ at $(1,-2,e^{-5})$.

A normal vector to the tangent plane is given by

$$< f_x(1,-2), f_y(1,-2), -1 > .$$

Thus at $(1,-2,e^{-5})$,

$f_x(x,y) = -2xe^{-x^2-y^2}, f_y(x,y) = -2ye^{-x^2-y^2} \Rightarrow \overrightarrow{\mathbf{n}} =< -2e^{-5}, 4e^{-5}, -1 >$

and hence the tangent plane is given by the equation

$-2e^{-5}(x-1)+4e^{-5}(y+2)-1(z-e^{-5}) = 0$ or $z = -2e^{-5}(x-1)+4e^{-5}(y+2)+$

Let $f(x, y) = e^{-x^2-y^2}$.

2. Use a linear approximation of $f(x, y)$ to approximate f at $(0.96, -1.97)$.

A good linear approximation of $f(x, y)$ to use to approximate f at $(0.96, -1.97)$ is the linear approximation at $(1, -2)$. Since the linear approximation is the same as the tangent plane, we already have the linear approximation we need:

$$L(x, y) = -2e^{-5}(x - 1) + 4e^{-5}(y + 2) + e^{-5}.$$

We can use this to approximate $f(0.96, -1.97)$:

$$\begin{aligned} f(0.96, -1.97) &\approx L(0.96, -1.97) = -2e^{-5}(0.96 - 1) + 4e^{-5}(-1.97 - (-2 \\ &= -.8e^{-5} + 1.2e^{-5} + e^{-5} = 1.4e^{-5} \approx 0.0094. \end{aligned}$$

For comparison purposes, from Maple I find that $f(0.96, -1.97) \approx 0.0082$.