Find the following directional derivatives:

1. $f(x, y) = 2e^{4x/y} - 2x$, $(a, b) = (2, -1)$, in the direction of $< 1, 3 >$
   - A unit vector in the direction of $< 1, 3 >$ is $\overrightarrow{\mathbf{u}} =< \frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}} >$.

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$$\begin{aligned} D_{\overrightarrow{\mathbf{u}}} f(2,-1) &= < f_x(2,-1), f_y(2,-1) > \cdot \overrightarrow{\mathbf{u}} \\ &= < f_x(2,-1), f_y(2,-1) > \cdot < \frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}} > \end{aligned}$$

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$$\begin{aligned} f_x(x,y) &= 2e^{4x/y} \cdot \frac{4}{y} - 2 \Longrightarrow f_x(2,-1) = -8e^{-8} - 2 \\ f_y(x,y) &= 2e^{4x/y} \cdot -\frac{4x}{y^2} \Longrightarrow f_y(2,-1) = -16e^{-8} \end{aligned}$$

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$$\begin{aligned} \Longrightarrow D_{\overrightarrow{\mathbf{u}}} f(2,-1) &= < -8e^{-8} - 2, -16^{-8} > \cdot < \frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}} > \\ &= \frac{-8e^{-8} - 2}{\sqrt{10}} - \frac{48e^{-8}}{\sqrt{10}} \end{aligned}$$

2. $f(x, y, z) = x^3yz^2 - 4xy$, $(a, b, c) = (1, -1, 2)$, in the direction of $< 2, 0, -1 >$.

A unit vector in the direction of $< 2, 0, -1 >$ is $\overrightarrow{\mathbf{u}} = \dfrac{< 2, 0, -1 >}{\sqrt{5}}$.

$$D_{\overrightarrow{\mathbf{u}}} f(1, -1, 2) = < f_x(1, -1, 2), f_y(1, -1, 2), f_z(1, -1, 2) > \cdot \frac{< 2, 0, -1 >}{\sqrt{5}}.$$

2. $f(x,y,z) = x^3yz^2 - 4xy$, $(a,b,c) = (1,-1,2)$, in the direction of $< 2, 0, -1 >$.

A unit vector in the direction of $< 2,0,-1 >$ is $\overrightarrow{\mathbf{u}} = \dfrac{< 2,0,-1 >}{\sqrt{5}}$.

$$D_{\overrightarrow{\mathbf{u}}} f(1,-1,2) =< f_x(1,-1,2), f_y(1,-1,2), f_z(1,-1,2) > \cdot \frac{< 2,0,-1 >}{\sqrt{5}}.$$

$$\begin{aligned}
f_x(x,y,z) &= 3x^2yz^2 - 4y \\
&\Longrightarrow f_x(1,-1,2) = 3(1)(-1)(4) + 4 = -8 \\
f_y(x,y,z) &= x^3z^2 - 4x \\
&\Longrightarrow f_y(1,-1,2) = (1)(4) - 4 = 0 \\
f_z(x,y,z) &= 2x^3yz \\
&\Longrightarrow f_z(1,-1,2) = 2(1)(-1)(2) = -4
\end{aligned}$$

$$\overrightarrow{\mathbf{u}} = \frac{< 2, 0, -1 >}{\sqrt{5}} \quad f_x(1, -1, 2) = -8 \quad f_y(1, -1, 2) = 0 \quad f_z(1, -1, 2) = -4$$

$$\Longrightarrow D_{\overrightarrow{\mathbf{u}}} f(1, -1, 2) = < -8, 0, -4 > \cdot \frac{< 2, 0, -1 >}{\sqrt{5}} = \frac{-16 + 4}{\sqrt{5}} = -\frac{12}{\sqrt{5}}.$$