Find the following directional derivatives:

1. $f(x,y) = 2e^{4x/y} - 2x$, $(a,b) = (2,-1)$, in the direction of $< 1, 3 >$

- A unit vector in the direction of $< 1, 3 >$ is $\vec{\mathbf{u}} =< \frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}} >$.
-

$$\begin{aligned} D_{\vec{\mathbf{u}}} f(2,-1) &= < f_x(2,-1), f_y(2,-1) > \cdot \vec{\mathbf{u}} \\ &= < f_x(2,-1), f_y(2,-1) > \cdot < \frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}} > \end{aligned}$$

$$\begin{aligned} f_x(x,y) &= 2e^{4x/y} \cdot \frac{4}{y} - 2 \Longrightarrow f_x(2,-1) = -8e^{-8} - 2 \\ f_y(x,y) &= 2e^{4x/y} \cdot -\frac{4x}{y^2} \Longrightarrow f_y(2,-1) = -16e^{-8} \end{aligned}$$

$$\begin{aligned} \Longrightarrow D_{\vec{\mathbf{u}}} f(2,-1) &= < -8e^{-8} - 2, -16^{-8} > \cdot < \frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}} > \\ &= \frac{-8e^{-8} - 2}{\sqrt{10}} - \frac{48e^{-8}}{\sqrt{10}} \end{aligned}$$

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$$\Longrightarrow D_{\overrightarrow{\mathbf{u}}} f(2,-1) = \overbrace{< -8e^{-8} - 2, -16^{-8} >}^{\nabla f(2,-1)} \cdot \overbrace{< \frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}} >}^{\overrightarrow{\mathbf{u}}}$$

2. $f(x,y,z) = x^3yz^2 - 4xy$, $(a,b,c) = (1,-1,2)$, in the direction of $< 2,0,-1 >$.

A unit vector in the direction of $< 2,0,-1 >$ is $\overrightarrow{\mathbf{u}} = \dfrac{< 2,0,-1 >}{\sqrt{5}}$.

$$D_{\overrightarrow{\mathbf{u}}} f(1,-1,2) =< f_x(1,-1,2), f_y(1,-1,2), f_z(1,-1,2) > \cdot \frac{< 2,0,-1 >}{\sqrt{5}}.$$

$$\begin{aligned}
f_x(x,y,z) &= 3x^2yz^2 - 4y \\
&\Longrightarrow f_x(1,-1,2) = 3(1)(-1)(4) + 4 = -8 \\
f_y(x,y,z) &= x^3z^2 - 4x \\
&\Longrightarrow f_y(1,-1,2) = (1)(4) - 4 = 0 \\
f_z(x,y,z) &= 2x^3yz \\
&\Longrightarrow f_z(1,-1,2) = 2(1)(-1)(2) = -4
\end{aligned}$$

2. $f(x,y,z) = x^3yz^2 - 4xy$, $(a,b,c) = (1,-1,2)$, in the direction of $< 2, 0, -1 >$.

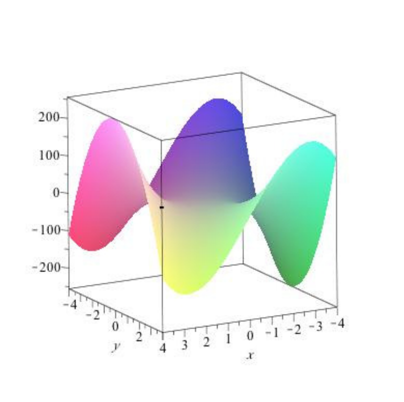
A unit vector in the direction of $< 2, 0, -1 >$ is $\overrightarrow{\mathbf{u}} = \frac{< 2, 0, -1 >}{\sqrt{5}}$.

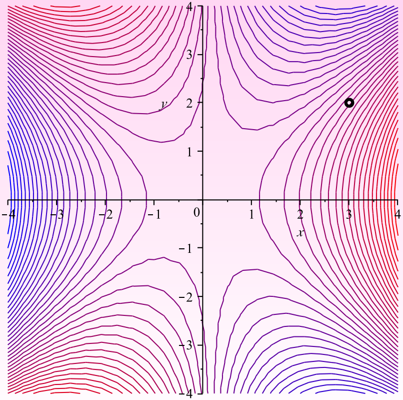
$$D_{\overrightarrow{\mathbf{u}}} f(1,-1,2) = \underbrace{< f_x(1,-1,2), f_y(1,-1,2), f_z(1,-1,2) >}_{\nabla f(1,-1,2)} \cdot \underbrace{\frac{< 2, 0, -1 >}{\sqrt{5}}}_{\overrightarrow{\mathbf{u}}}.$$

$$\begin{aligned} f_x(x,y,z) &= 3x^2yz^2 - 4y \\ &\Longrightarrow f_x(1,-1,2) = 3(1)(-1)(4) + 4 = -8 \\ f_y(x,y,z) &= x^3z^2 - 4x \\ &\Longrightarrow f_y(1,-1,2) = (1)(4) - 4 = 0 \\ f_z(x,y,z) &= 2x^3yz \\ &\Longrightarrow f_z(1,-1,2) = 2(1)(-1)(2) = -4 \end{aligned}$$

Consider

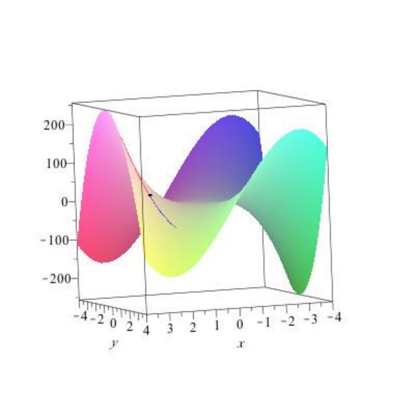
the surface $f(x, y) = 4x^3 - 6xy^2 + y^2$.
The point $(3, 2)$ is shown on both the graph of the surface and on the contour plot below.
In which direction should we head if we want to go up as steeply as possible?

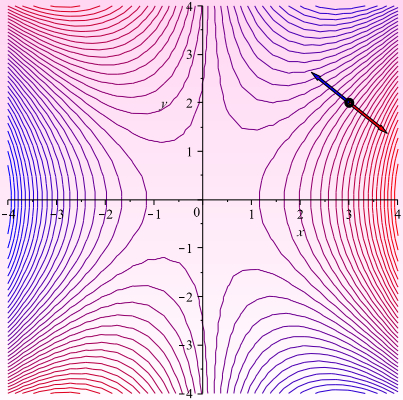




Consider

the surface $f(x, y) = 4x^3 - 6xy^2 + y^2$.
$\nabla f(3, 2)$ is shown in red and $-\nabla f(3, 2)$ is shown in blue on both the surface and the contour plot.
Do these look as if they point in the direction of steepest ascent and steepest descent?





We found that $\nabla f(3,2) =< 84, -68 >$. The claim therefore says that we should head in the direction $< 84, -68 >$ from the point $(3,2)$ to go up as steeply as possible, *and* should head in the direction $< -84, 68 >$ from that point to go down as steeply as possible.

Find the directions of maximum and minimum change of f at the given point, and the values of the maximum and minimum rates of change.

1. $f(x, y) = x^2 - y^3$, at the point $(-1, -2)$

2. $f(x, y, z) = 4x^2yz^3$, at the point $(1, 2, 1)$