1. Let $f(x, y) = \sqrt{x^2 + 4y^2}$.
   1.1 Identify the surface $z = f(x, y)$ by sketching traces or plotting it in Maple.
   1.2 On a single set of 2-dimensional axes, sketch the traces $z = 1$, $z = 2$, $z = 3$, and $z = 4$ to form a *contour plot* of the surface. Pay attention to the relationship between the contour plot and the surface.

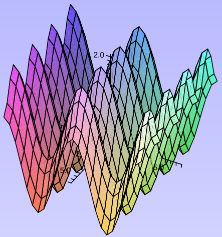
2. Match the functions to the surfaces:

(a) $f(x, y) = x^2 - \frac{x^4}{4} + y^2$

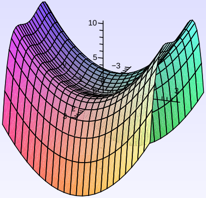
(b) $f(x, y) = (\sin(x))^2 + y^2$

(c) $f(x, y) = (\sin(x))^2 + \cos(y)$

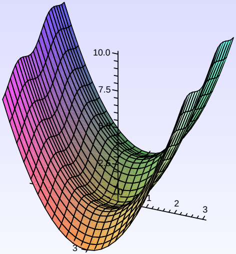
Surface A



Surface B

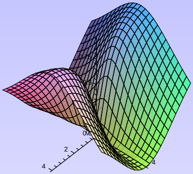


Surface C

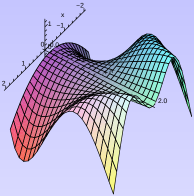


3. Match the surfaces to the contour plots

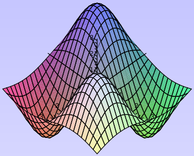
Surface (a)



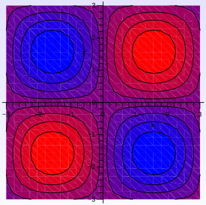
Surface (b)



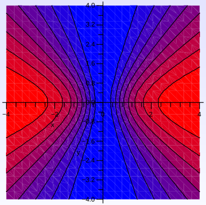
Surface (c)



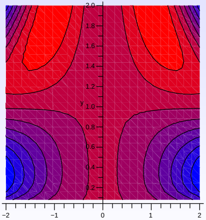
Contour Plot A



Contour Plot B



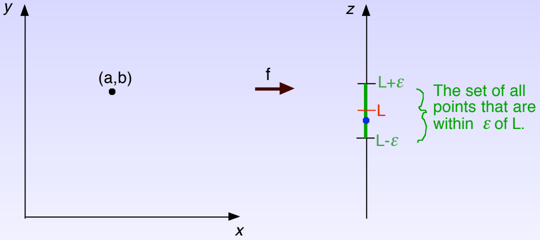
Contour Plot C



Idea behind the definition of the multivariate limit:

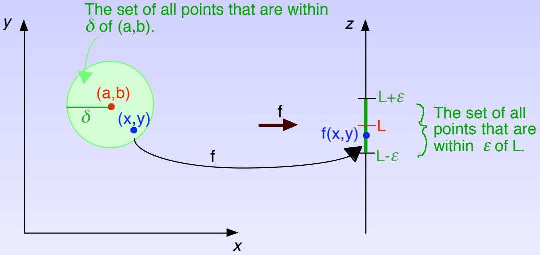
Given an element $(a, b) \in \mathbb{R}^2$, and a real number $L \in \mathbb{R}$. Does $\lim_{(x,y)\to(a,b)} f(x, y) = L$?

Let ϵ be an arbitrary positive real number. Mark off the set of all points within ϵ of L on the z-axis.



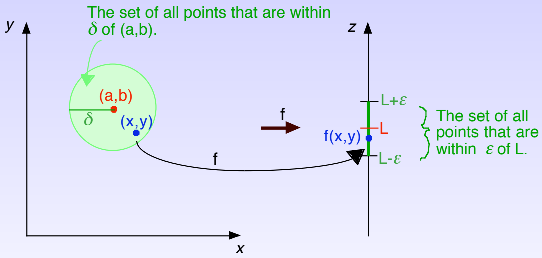
Idea behind the definition of the multivariate limit:

Does there exist a circle centered at (a, b), such that every point in that circle gets sent by f to the region around L that we marked off?



Idea behind the definition of the multivariate limit:

If, for *every* $\epsilon > 0$, there is a circle around (a, b) so that every point in the circle gets sent by f to the interval $(L - \epsilon, L + \epsilon)$, then we say that $\lim_{(x,y)\to(a,b)} f(x, y) = L$, because we can get arbitrarily close to L, just by choosing a small enough radius around (a, b).



Definition:

$\lim_{(x,y)\to(a,b)} f(x,y) = L$ if and only if for every $\epsilon > 0$ there exists a $\delta > 0$ such that

$$d\left((x,y),(a,b)\right) < \delta \qquad \Longrightarrow \qquad |f(x,y) - L| < \epsilon$$

Example:

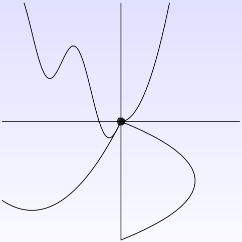
Let $z = f(x, y)$, where $f(x, y) = 1 - x^2 - y^2$. Because of your Calc 1 experience with limits, you will not be surprised that

$$\lim_{(x,y)\to(0,0)} f(x, y) = 1.$$

What that means:

No matter what path you look at that leads to $(0, 0)$, on the surface, that path approaches $z = 1$.

A few possible paths



The surface $z = 1 - x^2 - y^2$