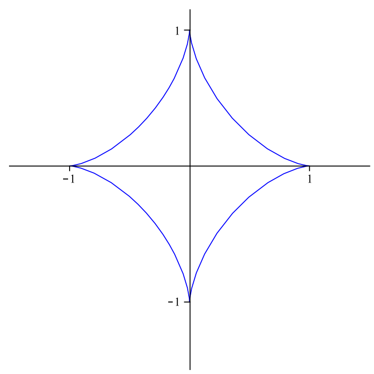
Smooth vector-valued functions, change of parameters, arclength, and the chain rule

1. The figure below shows the graph of the *four-cusped hypocycloid* $\vec{\mathbf{r}}(t) = \cos^3(t)\vec{\mathbf{i}} + \sin^3(t)\vec{\mathbf{j}}$ $(0 \le t \le 2\pi)$.



   (a) Give an informal explanation of why $\vec{\mathbf{r}}(t)$ is not smooth.

   (b) Confirm that $\vec{\mathbf{r}}(t)$ is not smooth by examining $\vec{\mathbf{r}}\,'(t)$ both graphically and algebraically.

2. *Illustrates that a change of parameter can transform a smooth vector-valued function to one that is not smooth - without changing the graph!*

   (a) Graph $\vec{\mathbf{r}}(t) = t\vec{\mathbf{i}} + t^2\vec{\mathbf{j}}$, $-1 \le t \le 1$, and explain why $\vec{\mathbf{r}}(t)$ is a smooth function of the parameter t, either using an algebraic or geometric argument.

   (b) Rewrite $\vec{\mathbf{r}}$ using the change of parameter $u = \sin(t)$.

   (c) Show that $\vec{\mathbf{r}}(u)$ is *not* a smooth function of the parameter u.

3. Consider the line given by $\vec{\mathbf{r}}(t) =< 1, 3, 4 > +t < 1, -2, 2 >$.

   (a) Find the arclength parametrization of this line that has the same direction, and has reference point $(1, 3, 4)$.

   (b) Use the new parametrization of this line that you found in part (a) to find the point on the line that is 25 units from the reference point in the direction of increasing parameter.

4. Find an arclength parametrization of the curve $\vec{\mathbf{r}}(t) = \sin(e^t)\vec{\mathbf{i}} + \cos(e^t)\vec{\mathbf{j}} + \sqrt{3}e^t\vec{\mathbf{k}}$, $t \ge 0$ that has the same orientation and has $t = 0$ as the reference point.