Where We're Headed:

- **Today:**
  - Look at the two problems you worked on at the end of class Monday; use them to briefly discuss conservative vector fields.
  - Integrating over curves rather than axes or regions (Line Integrals)
    - First, multivariate functions (area between a curve and a surface)
    - Next, vector fields (work)
- **Going forward:**
  - Fundamental Theorem for Line Integrals - Conservative Vector Fields and Independence of Path
  - Green's Theorem (which is also somewhat like a Fundamental Theorem of Line Integrals)

Solution: Problem 1, In Class Work from Last Time

1. Find a potential function $f(x, y)$ for the vector field $\overrightarrow{\mathbf{F}}(x, y) = < \cos y - 2 + y^2 e^{xy^2}, \cos y - x \sin y + 3y^2 + 2xye^{xy^2} >$.

$$f_x = \cos(y) - 2 + y^2 e^{xy^2} \quad \Rightarrow \quad f(x, y) = \int \cos(y) - 2 + y^2 e^{xy^2} \, dx$$

$$\Rightarrow \quad f(x, y) = x\cos(y) - 2x + e^{xy^2} + g(y)$$

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$$\Rightarrow \quad f_y = -x\sin(y) + 2yxe^{xy^2} + g'(y)$$

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$$\Rightarrow \quad f_y = -x\sin(y) + 2yxe^{xy^2} + g'(y)$$

Since we already know what f_y is, we can solve for $g(y)$:

$$f_y = \cos(y) - x\sin(y) + 3y^2 + 2xye^{xy^2} \quad \Rightarrow \quad g'(y) = \cos(y) + 3y^2$$

Solution: Problem 1, In Class Work from Last Time

1. Find a potential function $f(x,y)$ for the vector field
$\overrightarrow{\mathbf{F}}(x,y) =< \cos y - 2 + y^2 e^{xy^2}, \cos y - x\sin y + 3y^2 + 2xye^{xy^2} >$.

$$
\begin{aligned}
f_x = \cos(y) - 2 + y^2 e^{xy^2} \quad &\Rightarrow \quad f(x,y) = \int \cos(y) - 2 + y^2 e^{xy^2}\, dx \\
&\Rightarrow \quad f(x,y) = x\cos(y) - 2x + e^{xy^2} + g(y) \\
&\Rightarrow \quad f_y = -x\sin(y) + 2yxe^{xy^2} + g'(y)
\end{aligned}
$$

Since we already know what f_y is, we can solve for $g(y)$:

$$
\begin{aligned}
f_y = \cos(y) - x\sin(y) + 3y^2 + 2xye^{xy^2} \quad &\Rightarrow \quad g'(y) = \cos(y) + 3y^2 \\
&\Rightarrow \quad g(y) = \sin(y) + y^3 + c
\end{aligned}
$$

Thus *one* potential function for $\overrightarrow{\mathbf{F}}$ is

$$f(x,y) = x\cos(y) - 2x + e^{xy^2} + \sin(y) + y^3.$$

Problem 2, In Class Work from Last Time

2. Find a potential function $f(x, y)$ for the vector field

$$\overrightarrow{\mathbf{F}}(x, y) = < 2y + 2xy^2, 2x + 2y >$$

... if you can

Summarize Results

1. The vector field

$$\overrightarrow{\mathbf{F}}(x,y) =< \cos y - 2 + y^2 e^{xy^2}, \cos y - x \sin y + 3y^2 + 2xye^{xy^2} >$$

   has a potential function:

$$f(x,y) = x\cos(y) - 2x + e^{xy^2} + \sin(y) + y^3.$$

   That is, $\overrightarrow{\nabla} f = \overrightarrow{\mathbf{F}}$.

2. The vector field

$$\overrightarrow{\mathbf{F}}(x,y) =< 2y + 2xy^2, 2x + 2y >$$

   does **not** have any potential function.
   That is, there is no function $f(x,y)$ so that $\overrightarrow{\nabla} f = \overrightarrow{\mathbf{F}}$.

Summarize Results

1. The vector field

$$\overrightarrow{\mathbf{F}}(x,y) = < \cos y - 2 + y^2 e^{xy^2}, \cos y - x\sin y + 3y^2 + 2xye^{xy^2} >$$

has a potential function:

$$f(x,y) = x\cos(y) - 2x + e^{xy^2} + \sin(y) + y^3.$$

That is, $\overrightarrow{\nabla} f = \overrightarrow{\mathbf{F}}$. $\overrightarrow{\mathbf{F}}$ is a conservative vector field

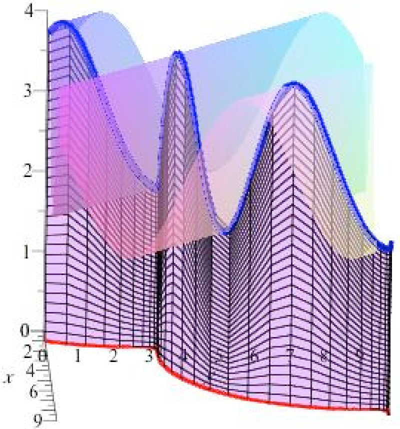
2. The vector field

$$\overrightarrow{\mathbf{F}}(x,y) = < 2y + 2xy^2, 2x + 2y >$$

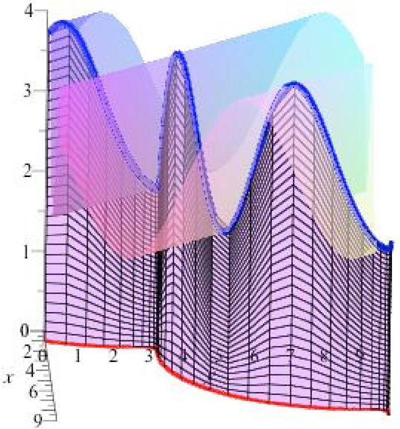
does **not** have any potential function.
That is, there is no function $f(x,y)$ so that $\overrightarrow{\nabla} f = \overrightarrow{\mathbf{F}}$.
$\overrightarrow{\mathbf{F}}$ is **not** a conservative vector field

Integrating over a curve $\mathcal{C}$

We want to find the area of the vertical surface that we create if we go straight from a curve $\mathcal{C}$ in the xy plane up to surface $z = f(x, y)$.



Area between curve $\mathcal{C}$ and surface $z = f(x, y)$



- Subdivide $\mathcal{C}$ into n sub-curves.
- Let ΔA_i = signed area of ith strip.
- Area $= \sum \Delta A_i$
- Approximate ΔA_i:
  Let $(x_i^\star, y_i^\star)$ be on ith sub-curve,
  Δs_i = arc length of ith sub curve.

$$\Delta A_i \approx f(x_i^\star, y_i^\star)\Delta s_i.$$

- $A \approx \displaystyle\sum_{i=1}^{n} f(x_i^\star, y_i^\star)\Delta s_i$
- $A = \displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^\star, y_i^\star)\Delta s_i$