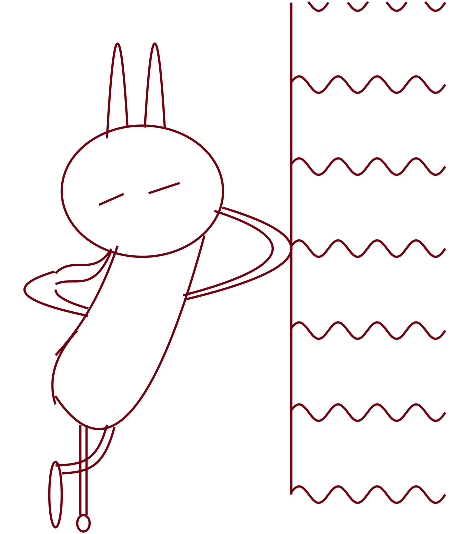
# WeBWorK Problem 1

Evaluate the line integral $\int_{\mathcal{C}} y\ dx + x\ dy$ where $\mathcal{C}$ is the parametrized path $x = t^2$, $y = t^3$, $3 \le t \le 7$.

$$\begin{aligned}
\int_{\mathcal{C}} y\ dx + x\ dy &= \int_{\mathcal{C}} < y, x > \cdot d\overrightarrow{\mathbf{r}} \\
&= \int_3^7 < y(t), x(t) > \cdot \overrightarrow{\mathbf{r}}\ '\ dt \\
&= \int_3^7 < t^3, t^2 > \cdot < 2t, 3t^2 > \ dt \\
&= \int_3^7 2t^4 + 3t^4\ dt \\
&= \int_3^7 5t^4\ dt = t^t \Big|_3^7 = 7^5 - 3^5
\end{aligned}$$

# Maple Projects



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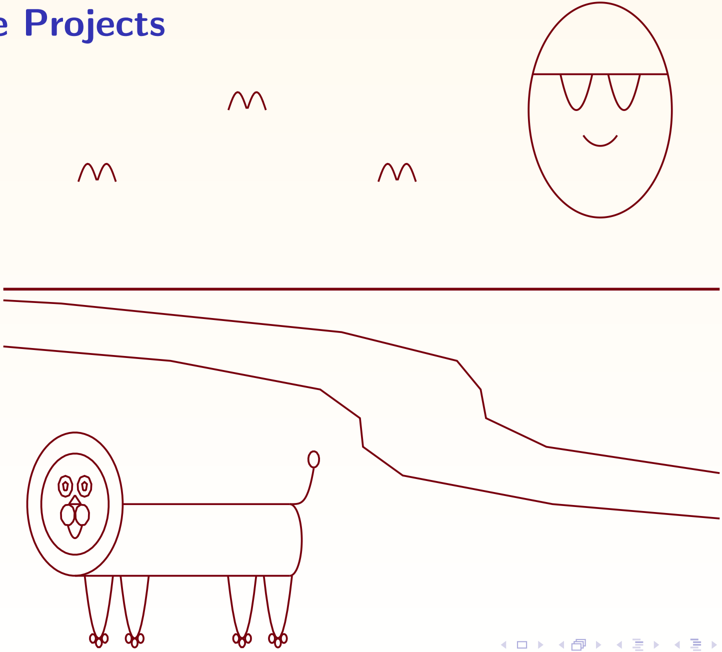

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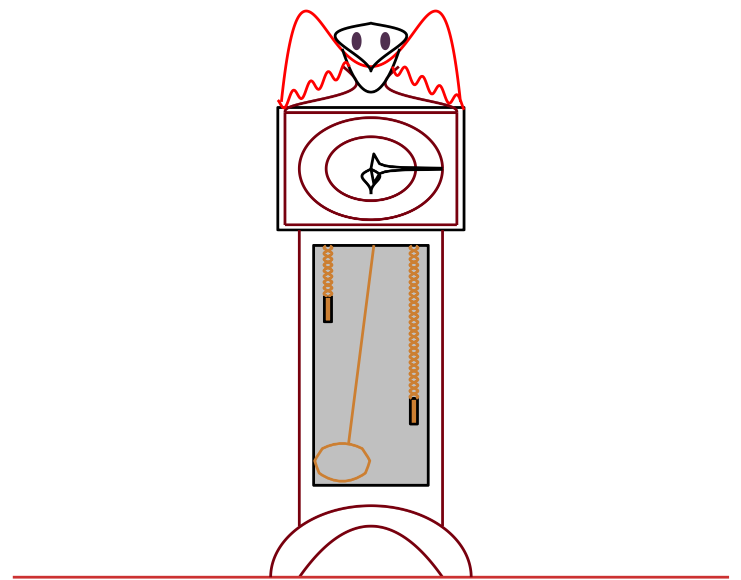

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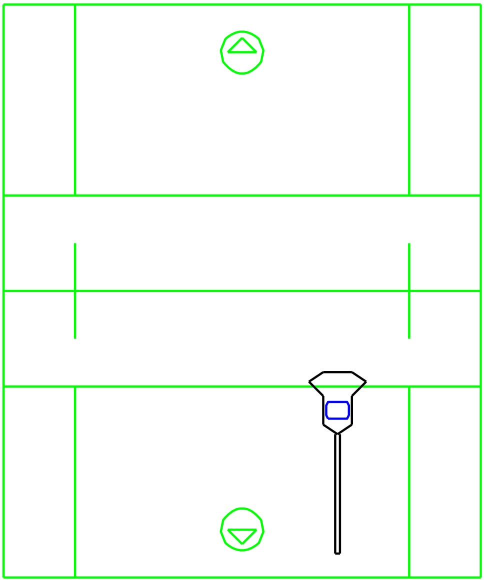

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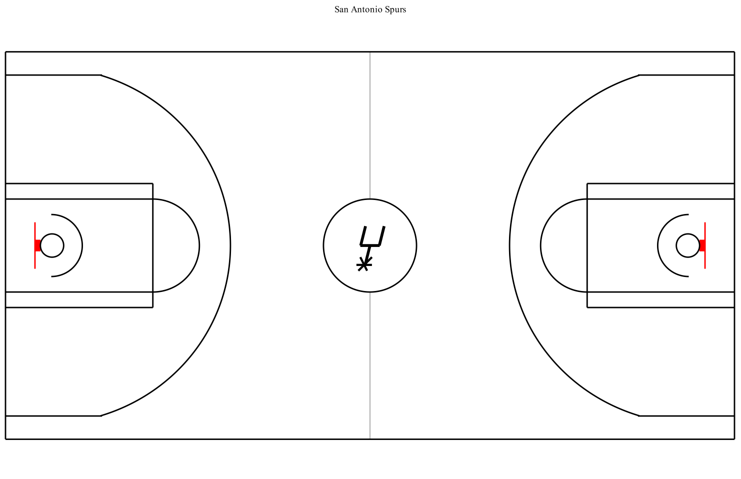

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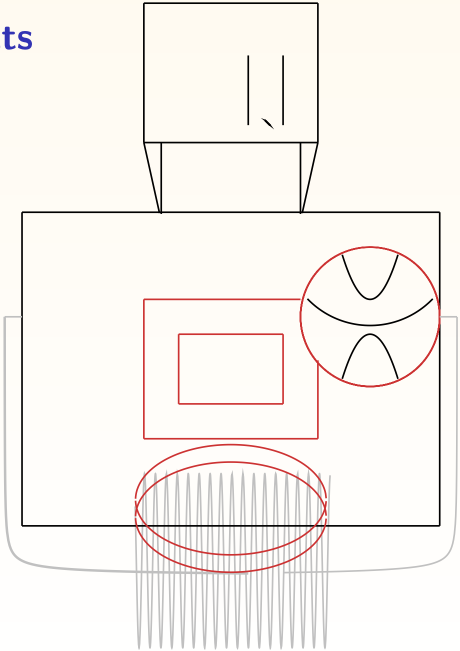

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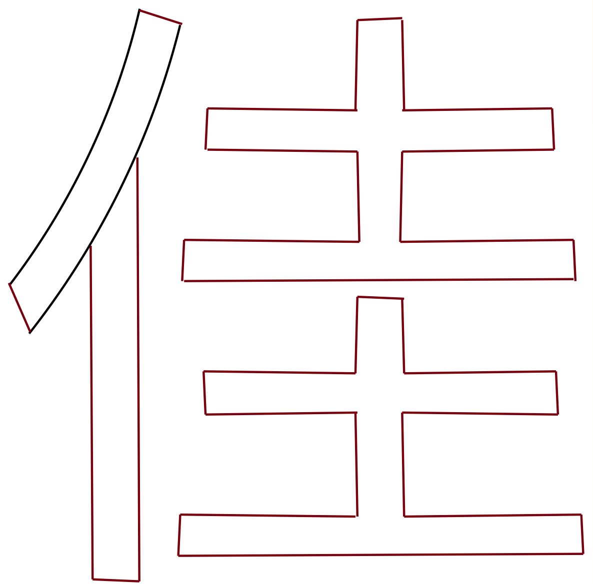

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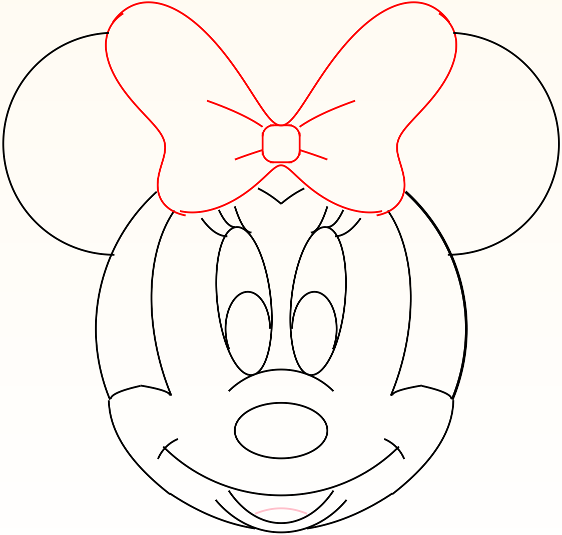

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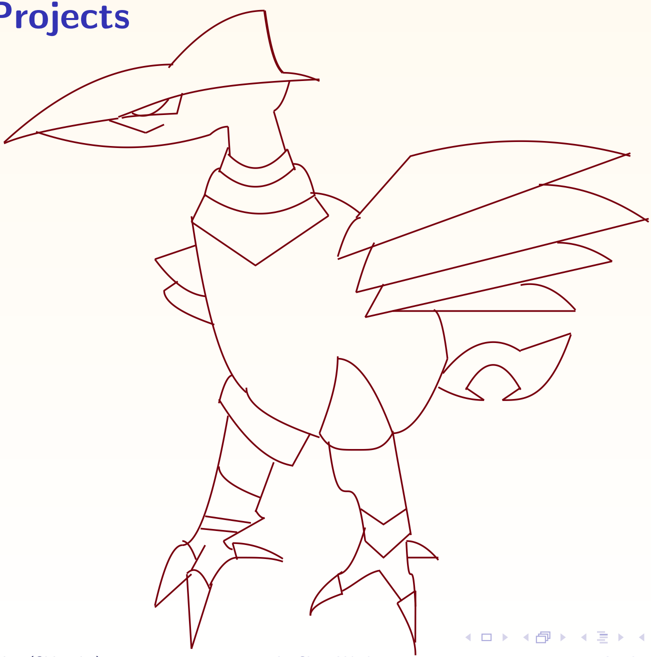


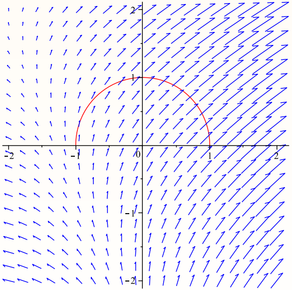
# Maple Projects

# Recall In Class Work from Last Class:

Let $\overrightarrow{\mathbf{F}}(x, y) =< 2x + y + 2, x + 3 >$, $\mathcal{C}_1$ be the upper unit semicircle oriented c.c., and $\mathcal{C}_2$ be the line segment from $(1, 0)$ to $-1, 0)$.

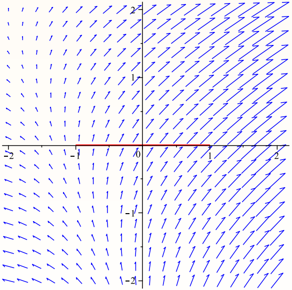
Work done by the force field $\overrightarrow{\mathbf{F}}$ acting on an object moving . . .

along $\mathcal{C}_1$:




$$W_1 = \int_{\mathcal{C}_1} \overrightarrow{\mathbf{F}} \cdot d\overrightarrow{\mathbf{r}} = -4$$

along $\mathcal{C}_2$:




$$W_2 = \int_{\mathcal{C}_2} \overrightarrow{\mathbf{F}} d\overrightarrow{\mathbf{r}} = -4$$

# Recall - Conservative Vector Fields

- If a scalar function $f : \mathbb{R}^n \to \mathbb{R}$ and a vector field $\overrightarrow{\mathbf{F}}$ are related by $\overrightarrow{\mathbf{F}} = \overrightarrow{\nabla} f$, then $f$ is called a **potential function** for $\overrightarrow{\mathbf{F}}$.
- If a vector field $\overrightarrow{\mathbf{F}}$ has a potential function, then $\overrightarrow{\mathbf{F}}$ is called a **conservative vector field**.
- The vector field we were just looking at,

  $$\overrightarrow{\mathbf{F}}(x, y) =< 2x + y + 2, x + 3 >,$$

  is conservative:

  If $f(x, y) = x^2 + xy + 2x + 3y$, then $\overrightarrow{\nabla} f =< 2x + y + 2, x + 3 >= \overrightarrow{\mathbf{F}}$.

# The Fundamental Theorem of Line Integrals

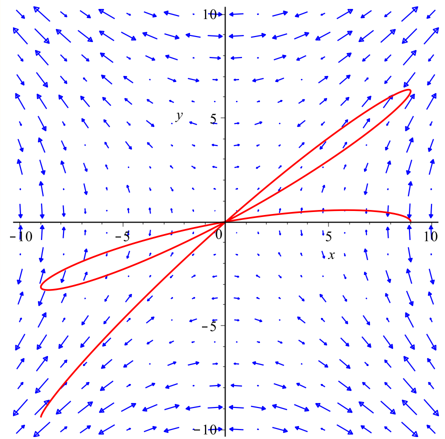
Suppose that $\overrightarrow{\mathbf{F}}(x, y)$ is a continuous vector field on the open, connected region $D \subset \mathbb{R}^2$, and let $\mathcal{C}$ be a piecewise-smooth curve in $D$, beginning at $(x_1, y_1)$ and ending at $(x_2, y_2)$.

If $\overrightarrow{\mathbf{F}}$ is conservative, with $\overrightarrow{\mathbf{F}}(x, y) = \overrightarrow{\nabla} f(x, y)$, then

$$\int_{\mathcal{C}} \overrightarrow{\mathbf{F}}(x, y) \cdot d\overrightarrow{\mathbf{r}} = f(x, y)\Big|_{(x_1, y_1)}^{(x_2, y_2)} = f(x_2, y_2) - f(x_1, y_1)$$

# Example:

Find the work done by the force field $\overrightarrow{\mathbf{F}} =< y\sin(xy), x\sin(xy) >$ acting on an object moving along the curve $\mathcal{C}$ given by
$\overrightarrow{\mathbf{r}}(t) =< 9\cos(t), t\cos(t) >$, $0 \le t \le 3\pi$.



# Example (continued)

Find the work done by the force field $\overrightarrow{\mathbf{F}} =< y\sin(xy), x\sin(xy) >$ acting on an object moving along the curve $\mathcal{C}$ given by
$\overrightarrow{\mathbf{r}}(t) =< 9\cos(t), t\cos(t) >$, $0 \leq t \leq 3\pi$.

Compare using the FToLI to using the methods from Section 14.2:

$$\begin{aligned}
W &= \int_{\mathcal{C}} \overrightarrow{\mathbf{F}} \cdot d\overrightarrow{\mathbf{r}} \\
&= \int_0^{3\pi} \left\langle t\cos(t)\sin\left(9\cos(t)t\cos(t)\right), 9\cos(t)\sin\left(9\cos(t)t\cos(t)\right) \right\rangle \\
&\qquad \cdot \left\langle -9\sin(t), -t\sin(t) + \cos(t) \right\rangle dt \\
&= \int_0^{3\pi} \left[ -9t\sin(t)\cos(t)\sin\left(9t\cos^2(t)\right) \right] \\
&\qquad + \left[ -9t\sin(t)\cos(t)\sin\left(9t\cos^2(t)\right) + 9\cos^2(t)\sin\left(9t\cos^2(t)\right) \right] dt
\end{aligned}$$