# In Class Work

1. Consider the surface $z = x^2 - y^2$.

 (a) Sketch the traces in the $yz$-plane, the $xz$-plane, the $xy$-plane and the planes $z = \pm 1$, $z = \pm 2$.

 (b) Use your traces to sketch a graph of the surface.

 (c) Check your sketch on Maple. Compare the following:

```
implicitplot3d(x^2-y^2-z=0,x=-3..3, y=-3..3, z=-3..3)
```

and

```
plot3d(x^2-y^2, x=-3..3, y=-3..3)
```

 This is a **hyperbolic paraboloid**.

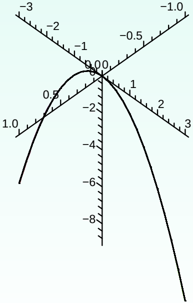
2. Find the equation of a **hyperboloid of two sheets** extending along the $y$-axis with vertices at $(0, 4, 0)$ and $(0, -4, 0)$. *(Review # 1 from last time)*

# Solutions

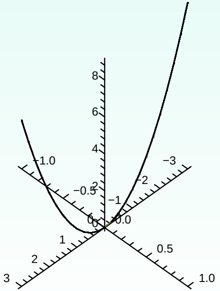
1. Consider the surface $z = x^2 - y^2$.

(a) Sketch the traces in the $yz$-plane, the $xz$-plane, the $xy$-plane and the planes $z = \pm 1$, $z = \pm 2$.

Trace in the $yz$-plane
$z = -y^2$: downward-opening parabola




Trace in the $xz$-plane
$z = x^2$: upward-opening parabola

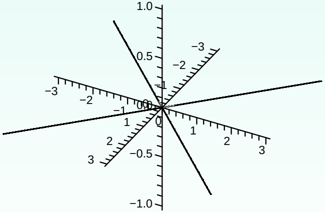


# Solutions

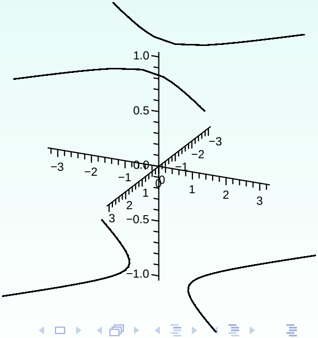
1. Consider the surface $z = x^2 - y^2$.

(a) Sketch the traces in the $yz$-plane, the $xz$-plane, the $xy$-plane and the planes $z = \pm 1$, $z = \pm 2$.

Trace in the $xy$-plane
$0 = x^2 - y^2 \implies y = \pm x$




Traces in $z = \pm 1$
$x^2 - y^2 = \pm 1$: hyperbolas

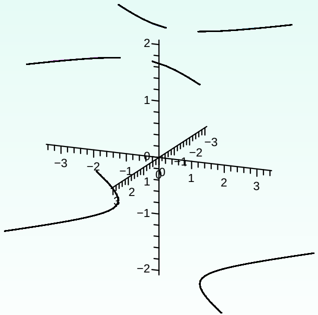


# Solutions

1. Consider the surface $z = x^2 - y^2$.

(a) Sketch the traces in the $yz$-plane, the $xz$-plane, the $xy$-plane and the planes $z = \pm 1$, $z = \pm 2$.

Traces in $z = \pm 2$: hyperbolas

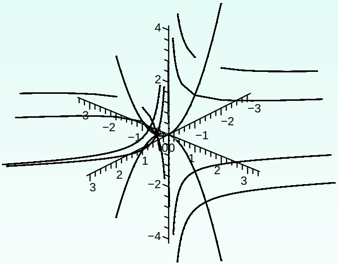


# Solutions

1. Consider the surface $z = x^2 - y^2$.

(b) Use your traces to sketch a graph of the surface.

Well, we can try to put the traces together ...



...but I, for one, feel like I had a better feel for what was going on before. **Hyperbolic paraboloids** are probably the hardest type of surface to sketch by hand!

# Solutions

2. Find the equation of a **hyperboloid of two sheets** extending along the $y$-axis with vertices at $(0,4,0)$ and $(0,-4,0)$.

- Saw Friday: $-x^2 - y^2 + z^2 = 1$ is a hyperboloid of 2 sheets opening along the $z$-axis, with circular cross-sections $\perp$ to the $z$-axis.
- Since we want the hyperboloid of two sheets to extend along the $y$ axis, the circular cross-sections will be $\perp$ to the y-axis, i.e. parallel to the xz-plane.
- Thus for (most) $y = k$, I'll have traces that look like $x^2 + z^2 = C$.

  (Since this is a hyperboloid in 2 sheets, there will be the gap in between the two sheets. For all of the $y$ where the gap occurs, there will be **no** trace.)

# Solutions

2. (continued)

- For the vertices to be at $(0, 4, 0)$ and $(0, -4, 0)$, I can't have any solutions for $x$ and $z$ that make sense when $y$ is in $(-4, 4)$.
- *That* means that the $x$ and $y$ terms will be negative: something like $y^2 - x^2 - z^2 = d$, where when $-4 < y < 4$, we end up with $-(x^2 + z^2) =$ a positive number.
- When $x = z = 0$, I need $y = \pm 4$.
- So one option is $y^2 - x^2 - z^2 = 16$.

# Functions of Two Variables

**Example:** Below are the graphs of four surfaces. Two of them are the graphs of $f_1(x, y) = [(x^2 + y^2) - 2]^3$ and $f_2(x, y) = y^2 \sin(x)$. Which two?