Time: 5:30-8:30, Thursday 4/10
Location: SC 1313, SC 1314
What It Covers: Mainly Sections 11.2, 11.3, 10.6, 12.1-12.6, and the beginning of 12.7. (Through the Second Derivatives Test). Note that because new ideas rely on old, some material from the last exam is sure to be on this one

- As with the first exam,
  - I'll aim for the exam to take 1 1/2 - 2 hours, but you may use 3.
  - You may handwrite one sheet of notes, on **one** side of an 81/2 $\times$ 11 sheet of paper. Include any definitions, theorems, or examples you like.
  - **During the exam, all electronics must be off and away.**
  - You will not have access to phone or computer, so bring a separate calculator.
  - You may **not** use your calculator to solve equations, do any Calculus, or to graph.
  - Anything covered in your reading, homework, or this study guide is fair game.
  - The exam can't cover everything, but the point is that you learn it all, well.
  - Remember: studying should give you a deeper, more connected understanding of the material; exam questions aim to see if you have reached that next level of understanding, and so won't be just like homework.
- Below, I have a list of some topics you should know.
- Following that list, I have included some sample problems to give you a sense of the style of question that may be on the exam. These do not cover every topic that maybe covered on the exam, so doing these should not constitute the entirety of your studying.
- There are additional problems on WeBWorK, under the name Study Guide 2.

Topics you should know:

1. Still need to know from before last exam: many things. Here are a few:
   - vector basics, including magnitude, dot product, and cross-product.
     - Part of this is knowing how dot and cross product are related to $\cos(\theta)$ and $\sin(\theta)$.
   - How to find parametric equations of lines and equations of planes
2. $\vec{\mathbf{F}} = m\vec{\mathbf{a}}$
3. How to differentiate and integrate vector-valued functions, and how to (partially) differentiate multivariate functions without technology. This includes finding second derivatives, second partials, mixed partials, and requires remembering the product rule, quotient rule, and chain rule from Calc 1. See §11.2, Problems 13-18 and 23-32; §12.3 Problems 1-16 as necessary.

4. The relationships between acceleration, velocity and position vectors
5. How to sketch the traces of a surface parallel to the coordinate planes; how to draw level curves of a function of two variables; the distinction between drawing traces and drawing a contour plot.
6. Be able to recognize a quadric surface (ellipsoid, elliptic paraboloid, hyperbolic paraboloid, etc) from an equation, or vice versa.
7. Matching a function of two variables with a graph of its surface, or with the graph of its contour plot. Also matching the graphs of surfaces with the graphs of their contour plots.
8. Identifying where local extrema and saddle points are from a contour plot.
9. Taking the limit of a function of two variables along a specific path, and *choosing* paths that go through the limiting point to take the limit along.
10. What is required for a limit of a function of 2 variables to exist
11. How to show a limit does *not* exist
12. Using the Squeeze Principle to show that a limit does exist
13. Continuity, and when you can evaluate a limit at a point simply by evaluating f at that point.
14. The chain rule for functions of several variables (particularly using tree diagrams to figure out how to state the particular version of the chain rule that applies to any given composite function.)
15. How to implicitly differentiate a function in two or more variables.
16. Finding the equation of the tangent plane to a surface, whether it's explicitly defined (Section 12.4, 12.6) or implicitly defined (Section 12.5, 12.6). You will also need to remember what a normal vector is, and how to find parametric equations of normal lines.
17. The gradient and what it **is** (not only that it's a vector, and how to find it, but the two things you should *always* associate with the gradient, according to your text.)
18. Finding the direction and rate of steepest ascent (or steepest descent), given a function
19. Sketching the path of steepest ascent on a contour plot
20. Directional derivatives, including their relationship to the gradient and that if $D_{\vec{\mathbf{u}}}f(a, b) = 0$, then $\vec{\mathbf{u}}$ points in the direction of the level curve through (a, b).
21. Finding critical points and classifying them using the Second Derivatives Test

Sample problems:

1. Do the WeBWorK *Study Guide 2* problems

2. Find all values of t so that $\vec{\mathbf{r}}\,'(t)$ is parallel to the xz-plane, if $\vec{\mathbf{r}}(t) = \left\langle t^2, \sin(t), t \right\rangle$

3. Suppose you are playing tennis on Mars, where the force of gravity is 38% of earth's (that is, 12.20 f/s^2). To take into account the weaker gravitational pull, you lengthen the court and raise the net. On your martian court, for a tennis serve to count (to be "in"), the ball must land before the service line 84 feet away from the server, and before that, must clear a net that is 50 feet away from the server and 4.5 feet high.

   On a serve, you strike the tennis ball horizontally from a height of 7.3 feet with initial speed 80 feet per second. Find a vector function for the position of the ball and determine whether the serve is in or not. If it is not in, how you could readjust the court design so that it *is*?

   Be sure that you are able to do this problem beginning only with the acceleration vector.

   For the surface $x^2 + y^2 - 9z^2 = -9$, first sketch appropriate traces (all on one set of 3D axes), and then put them together into a sketch of the surface. Identify the surface.

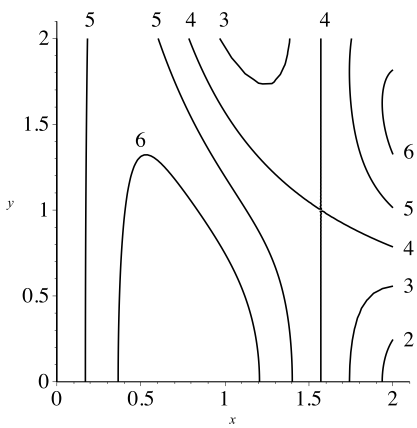
4. The temperature T (in °C) at any point in the region $-10 \le x \le 10, -10 \le y \le 10$ is given by the function
$$T(x, y) = 100 - x^2 - y^2.$$
   (a) Sketch isothermal curves (that is, curves of constant temperature) for $T = 100$°C, $T = 75$°C, $T = 50$°C, $T = 25$°C, and $T = 0$°C.
   (b) Suppose a heat-sealing bug is placed at each of the following points on the xy-plane. Thinking of the positive y-axis as north and the positive x-axis as east, what direction should the bug move in to increase its temperature fastest, if it is initially placed at the point
      i. $(-5, 5)$
      ii. $(7.5, 0)$
      iii. $(0, -7.5)$

5. Show that the following limit does exist *or* show that it does not exist and explain why.
$$\lim_{(x,y)\to(0,0)} \frac{x^2(y^2+2) + y(x^3+2y)}{x^2+y^2}.$$

6. If $f(x, y, z) = \cos(x\sin(z)) + e^{z+(xz)^4} + xy^2z$, find f_{xzy}.

   Note: You could do this problem a long way, or a shorter way.

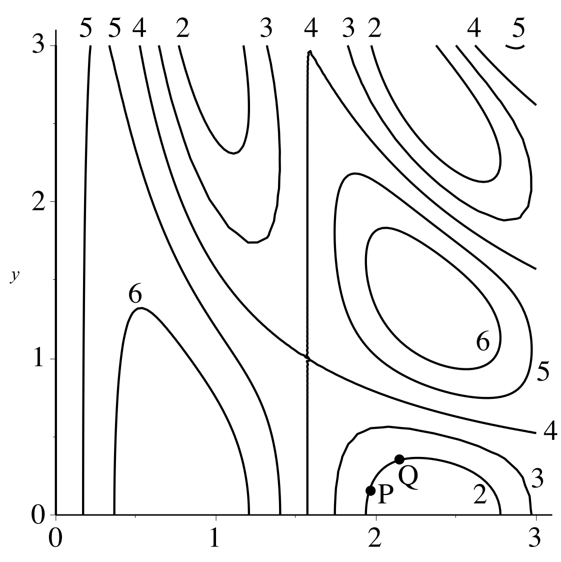
7. Use the given contour plot to estimate the linear approximation of $f(x, y)$ at the point $(1.4, 0)$. Then use that linear approximation to estimate $f(1.3, 0.1)$.



8. Use implicit differentiation to find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ if $e^z = xyz$.

9. The lengths of the sides and angles of a triangle are changing in a way so that that the area of the triangle remains constant. [Note that the triangle will not generally be a right triangle.] If one side is increasing at a rate of 3 cm/s and the second side is decreasing at a rate 2 cm/s, at what rate does the angle between those two sides change when the first side is 20 cm long and the second side is 30 cm, and the angle between them is $\pi/6$?

10. On the given contour plot, sketch in the paths of steepest ascent from the points P and Q. Pay attention to your results – are they expected, surprising?



11. Decide whether each of the following is true or false. If your answer is true, explain why. If your answer is false, give a counter-example (or, if it just doesn't make any sense at all, explain why).

    (a) In an equation of a quadric surface, if one variable is linear and the other two are squared, then the surface is a paraboloid wrapping around the axis corresponding to the linear variable.

    (b) Quadric surfaces are examples of graphs of functions of two variables.

    (c) The derivative of a vector-valued function gives the slope of the tangent line.

    (d) $\dfrac{\partial f}{\partial x}(a,b)$ gives the slope of the line tangent to $z = f(x,y)$ at (a,b).

    (e) If $\vec{\mathbf{u}} =< u_1, u_2 >$ is a unit vector and $D_{\vec{\mathbf{u}}} f(a,b) < 0$ then $f(a+u_1, b+u_2) < f(a,b)$.