Time: 2:00-5:00, Wednesday 5/8
Location: Come to SC 1314. Exam will be in SC 1314 and SC 1349
What it covers: Anything we've learned this semester is fair game

- Length: The final may contain more problems, or more complex problems. You (should) know the older material at a deeper level than you did for the two midterms, able to work more quickly through problems similar to ones you've seen before, and also able to put together ideas and work through new types of problems. Whereas I expected the first two exams to take a well-prepared (and well-rested) student about 1 1/2 - 2 hours, I would expect this one to take a well-prepared and well-rested student longer.
- You may again handwrite one $81/2 \times 11$ sheet of notes, **but** this time you may use both sides of the paper.
- **As always:**
  - All electronics that attach to the internet or to other people must be off and away.
    - You will (probably) need a calculator. Bring one that abides by this restriction.
  - You may **not** use your calculator to solve equations, do any Calculus, or to graph.
  - Anything covered in your reading, homework, or this study guide is fair game.
  - The exam can't cover everything, but the point is that you learn it all, well.
- Remember: by the time you're taking the final, you should have a deep and broad understanding of the material. Final questions may consequently be not only different from homework questions but also from midterm exam questions.
- Use previous study guides to study material from before the last midterm exam. This study guide only covers the newest material.
- Because the most recent material uses so many of the concepts from the beginning of the term, it is possible that I may choose to have many or even most of the problems cover the newest material, as that way a single question may organically require a lot of early topics as well. (For instance, to do one line integral problem, you may need to parametrize a curve, while to do another you may need to know take partial derivatives and also how to set up and evaluate a double integral.)
- Below, I have a list of some of the most recent topics you should know.
- Following that list, I have included some sample problems to give you a sense of the style of question covering the new material that may be on the exam. These do not cover every new topic that maybe covered on the exam, and of course the final will also cover the older material, so please do not study only from this.

- There are additional problems on WeBWorK, under the name Study Guide 3. I have also reopened study guides 1 and 2. I intentionally did this in a way that would erase your previous results – I thought that would be more useful.
- **Be aware:** Because so much of the material since the last exam is integration, it is quite possible that you will need to integrate (technology-free) on the exam, so be sure you're comfortable with integration by parts and u-substitution.

Topics you should know:

1. Every topic listed on the first two study guides

   and

2. The Extreme Value Theorem, and how to use the conclusions of the EVT to find the locations and values of absolute extrema over a closed and bounded region.
3. Know how to evaluate double integrals over rectangles.
4. The connection between double integrals and (signed) volume.
5. What Fubini's Theorem does for us
6. Finding limits of integration for a double integral over any reasonable region
7. Being able to switch the order of integration
8. Evaluating double integrals, once they're set up.
9. How to set up (and evaluate) a double integral to give the **area** of a region.
10. Using double integrals to find the volume bounded by two (or more) surfaces).
11. Using double integrals to find mass and center of mass.
12. Setting up the double integral to find surface area.
13. Matching a vector field with its graph
14. Determining whether a vector field is conservative and finding a potential function, whether $\vec{\mathbf{F}} : \mathbb{R}^2 \to \mathbb{R}^2$ or $\vec{\mathbf{F}} : \mathbb{R}^3 \to \mathbb{R}^3$.
15. Understanding the graphical relationships between a conservative vector field, its potential function, and the level curves of the potential function
16. Knowing line integrals with respect to arclength are used to find (vertical) area above or below a curve (like drapes), while component-wise line integrals are used to find work done by a force field in moving an object along a path.

17. Knowing what types of functions we use in each kind of line integral.
18. Setting up and evaluating a line integral with respect to arclength
19. Setting up and evaluating a component-wise line integral
20. Be able to determine from a graph whether the work done by a force field to move a particle along its path is positive or negative.
21. Know how (and when) to use the Fundamental Theorem of Line Integrals.
22. Know the statement of Green's Theorem and be able to determine whether it applies and if it does, how to use it.

Sample Problems

1. Do the WeBWorK *Study guide 3* problems – there are some ideas I'm not putting on this but that are covered there.

2. Go back and redo (or do) the previous study guides, both hand-out (available on main public webpage) and WeBWorK.

3. Find the absolute maximum and minimum values of
$$f(x, y) = 4xy^2 - x^2y^2 - xy^3$$
over the region D, where D is the closed triangular region in the xy-plane with vertices $(0, 0)$ $(6, 0)$, and $(0, 6)$.

4. Set up an integral or sum of integrals which will give the volume of the solid bounded by the $z = 2x+y-1$, $z = -2x$, $x = y^2$ and $x = 1$, then use Maple or other technology to compute.

   Note: This is almost identical to a homework problem. However, the change from $z = 2x + y + 1$ to $z = 2x + y - 1$ changes everything. Be very very careful that you know which plane is on top and which plane is on the bottom, and if necessary, break it up in to two or more volume integrals.

5. Find the area of the surface $z = \frac{2}{3}\left(\sqrt{2}x^{3/2} + y^{3/2}\right)$ over the region R bounded by $y = x^2$ and $y = x$, *without* technology.

6. Find the mass of a spring in the shape of a helix defined parametrically by $\mathcal{C} : x = t$, $y = 3\cos(t)$, $z = 3\sin(t)$ for $0 \leq t \leq \pi$ with density function $\rho(x, y, z) = yz\cos(x)$ by evaluating the line integral $\int_{\mathcal{C}} \rho(x, y, z)ds$. Do not use technology to evaluate the integral.

7. Determine whether the Fundamental Theorem of Line Integrals applies to each of the following line integrals. If it does, evaluate the integral. If it does not, set up the best way you can think of to find the value of the integral.

   (a) $\int_{\mathcal{C}} x^2y\,dx - xy^2\,dy$ over the counter-clockwise circle $\mathcal{C}$: $x^2 + y^2 = 4$

   (b) $\int_{\mathcal{C}} \left\langle e^x\cos(y), e^x\sin(y)\right\rangle \cdot d\vec{\mathbf{r}}$ over the line segment joining $(0,0)$ to (π,π)

   (c) $\int_{\mathcal{C}} 4x^3y^2 - 2xy^3\ dx + 2x^4y - 3x^2y^2 + 4y^3\ dy$ over the curve $\mathcal{C}$: $\vec{\mathbf{r}}(t) = \left\langle t + \sin(\pi t), 2t + \cos(\pi t)\right\rangle$, with $0 \le t \le 1$

8. Determine whether the following statements are true or false. If it is true, explain why. If it is false, explain why or give an example that disproves the statement.

   (a) If a continuous function $f(x,y)$ has two local maxima, then f must have a local minimum as well.

   (b) If a continuous function $f(x,y)$ has an absolute maximum on a closed and bounded region R, then it must have an absolute minimum on that region as well.

   (c) $\int\int_R f(x,y)\,dA$ gives the volume between $z = f(x,y)$ and the xy-plane.

   (d) $\int_0^1\int_0^x \sqrt{x+y^2}\,dy\,dx = \int_0^x\int_0^1 \sqrt{x+y^2}\,dx\,dy$

   (e) $\int_1^2\int_3^4 x^2e^y\,dy\,dx = \int_1^2 x^2\,dx\ \int_3^4 e^y\,dy$

   (f) The graph of a vector field shows vectors $\vec{\mathbf{F}}(x,y)$ for all points (x,y).

   (g) The line integral $\int_{\mathcal{C}} f(x,y)\ ds$ represents the surface area extending from $\mathcal{C}$ in the xy-plane to the surface $z = f(x,y)$.

   (h) There exists a function f with continuous second-order partial derivatives such that $f_x(x,y) = x + y^2$ and $f_y(x,y) = x - y^2$.

   (i) The work done by a conservative vector field in moving an object around a closed path is 0.