You will each adopt a group and apply to it the ideas we learn during the semester, as well as researching other information relevant to your group (historical context, applications, interpretations, for example). You will then discuss those results in a paper, and later summarize the results in a poster for a class poster session. My hope with this assignment is that having a specific group to investigate whenever we learn new concepts and properties will help you to grasp the concepts more deeply. Furthermore, I hope you become more comfortable with math research – figuring out what questions you need to ask, and then answering them.

CONCEPTS TO INVESTIGATE:
I am including below a list of algebraic properties and concepts for you to investigate. (Of course, you'll learn the vocabulary as we go along.) Each adopted group will vary as to which properties are easy to investigate and which are harder; you may not be able to develop information about some of the concepts, but you may be able to find information not referred to on this list. Please don't assume any of the items on this list are irrelevant to your group until you discuss them with me.

- Verify your example **is** a group. This should of course be preceded by whatever definitions and explanations of the elements and operation are necessary.
- The order of the group
- Order of each element in the group
- Interpretations(s), applications of the group
- Cyclic?
- Generators? (This should be addressed whether or not the group is cyclic)
- Abelian?
- Subgroups: if possible (and sensible) give a complete subgroup lattice or a description of all subgroups; if neither of these is practical, then give and discuss an interesting collection.
- Find the center
- Find the centralizer of each element. (This may or may not be possible for infinite groups; if it's not possible, investigate as far as you can and draw whatever conclusions are possible.)
- Isomorphisms, automorphisms, and inner automorphisms. If practical, investigate the automorphism group and the inner automorphism group.
- Normal subgroups
- Quotient (factor) groups

WRITING THE PAPER:
In your paper, you should include all of the information you find on the above properties and concepts, as well as anything else you learn about your group along the way. Your paper should not be a collection of facts, or even a list of results; I'm sure you want to interest and inform us, so make your paper clear and engaging – mathematical writing doesn't have to put the audience to sleep or mentally exhaust the audience–assume your audience consists of your classmates. Writing about math in a clear, straightforward, and even eloquent way **is** possible, but I know it is difficult. I am happy to help you out, so come to me as often as you need to, and take advantage of your classmates' good nature also.

Your paper should include a title page and an abstract. You should consult other sources (at least one is required) for information not in our textbook. These sources should of course be properly referred to in the body of the text and in a bibliography. Expect your paper to be a minimum of 5-7 pages; it may well end up being 10 pages or more.

GRADING THE PAPER:

The criteria I'll use to grade your paper can be summarized as follows:

- **Content: correctness, completeness, and depth:**
  The point of your paper, of course, is the presentation of the results of your investigations. These include finding and proving or demonstrating all the basic properties of your group, determining additional properties which only your group may have, and possibly finding and presenting historical contexts, applications, or interpretations of your group. Your findings should be correct, you should discover everything you reasonably can about your group, and you should investigate in whatever depth seems called for in each particular case.

- **Clarity:**
  While the mathematical results are the point of the paper, there's no purpose to writing a paper if the results are not presented clearly. Remember, it is not the reader's job to understand what you are writing, it is your job to make yourself easily understood. To be understood, your writing must be clear on two levels.

  On the "global" level, you should pay attention to how the whole paper flows. Ask yourself in which order you should present the concepts–should you discuss whether the group is cyclic before or after you discuss whether the group is Abelian? Should you give an example before or after you give a proof? Make sure you are consistent in your notation and in your treatment of the subject. Are your arguments convincing?

  On the "local" level, you should pay attention to each paragraph. You should strive for clarity in each phrase, sentence, argument, figure, table, etc. Are the ideas in a paragraph all related, and are they in the order that best conveys your point? Is it clear to the reader where each logical step comes from? When you change trains of thought, do you begin a new paragraph? Are the figures and tables referred to at the appropriate time, and are they easily found by the reader?

- **Grammar, Punctuation, and Neatness:**
  Finally, while clarity is key to how much a person enjoys reading your paper, it is also important that they believe your results, and for that matter, even pay attention while they are reading. Clarity is an important factor in these issues, but so too is neatness, punctuation, and grammar. A paper that is visually a mess does not inspire confidence in the author, and neither do errors in grammar or punctuation. Worse even than dispelling confidence in your results, though, messiness, poor punctuation and grammar can be confusing and even misleading. Make sure the paper is neat and grammatically correct, and people will trust that you have put the same care into your results.

Note: For any given property that I've asked you each to investigate, the difficulty of making arguments or finding additional information will vary from adopted group to adopted group. Because I'm aware of this, I may well consider a paper with fewer but more difficult to ascertain facts to be worth as much or more as a paper with more but easy to ascertain facts. If you feel that many of the facts in your paper were easy to ascertain (for instance, if your group is Abelian and of a given order), you may want to investigate other aspects of your group in more depth than I would expect from a student whose group posed more difficulties. So when it comes time to select your group, don't fret that some groups are easier than others.

POSTER SESSION:
The last day or two of class, we will have a poster session. This involves each of you making a poster highlighting your findings, and taking a short time (3-4 minutes only, don't panic) to discuss your poster. Poster sessions are usually informal, consisting of short presentations on each poster, followed by time where you all roam about looking at the various posters and asking each other questions on your various groups, comparing results.

The goal of your poster is to communicate the nature and significance of your group. You should include (i) a statement of the questions you investigated and the results you found, (ii) a discussion of the significance of your findings and how it fits into broader questions in math, (iii) some indication of the techniques used in attacking your problem. A poster can not possibly contain all the thought that went into your findings, of course, so you will need to use your judgment to determine which techniques are worthy of mention, and how much detail to go into.

As we get closer to the actual time, I will give you a handout with advice on how to make an interesting poster; in the meantime, I suggest you take note as you wander throughout the science center, as there are many student posters on display here. What grabs your attention? Which posters do you walk right by without a second glance, and why? How important is the size of the writing? How important is color?

GRADING THE POSTER SESSION:
My main concerns during the poster session will be with your presentation of the material, both orally and visually. Strive to be clear, and engaging.

TIME SCHEDULE:

September 11	Lottery to adopt group (we have to do something that day, I suppose)
September 20	Short paper due, verifying your example **is** a group.
September 21-October 10	Research and develop information about your group. During this time, you are required to schedule at least one meeting with me to discuss your progress, and please feel free to come see me frequently. Bring **all** your notes and references whenever you come to see me.
October 11	Short paper due, explaining something interesting and something confusing about your group.
October 12-October 24	Continue researching your group.
October 25	First draft of paper due. Peer review takes place. **Please note:** At that point, we will not have learned all the material necessary to address all the topics I have suggested. You should be able to discuss everything except for normal subgroups and factor groups. Depending on how far we get, you eventually may be able to say more about isomorphisms, as well.
October 26-October 31	Respond to the suggestions you received. Refine your paper further. Investigate your group's cosets, if you haven't yet.
November 1	Second draft of paper due to me. **Please note:** We may still not have learned all the necessary material. I will review the draft, making suggestions for revisions, and conceivably also suggesting additional work. I may ask you to turn in a blank cassette tape as well, if I decide to dictate comments to you.
November 2-November 14	Investigate the normal subgroups and factor groups. Respond to my suggestions, and refine your paper still more.
November 15	Final draft of the paper due.
December 11	First day of poster session.

ADOPTABLE GROUPS

D_6	The Dihedral Group of order 12
$\mathbb{C}_{10}$	The 10 roots of unity with complex number multiplication
$\mathbb{Q}_{2,3}$	The set of all rational numbers of the form $2^m 3^n$ with m and n integers under ordinary integer multiplication.
Q_4	The Quaternion group
—	All groups of order 8
—	All groups of order 9
$U(23)$	The set of integers less than and relatively prime to 23 under multiplication modulo 23
$U(24)$	The set of integers less than and relatively prime to 24 under multiplication modulo 24
M	The multiplicative group of 2×2 matrices of the form $\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$ where x is a real number.
$GL(2, \mathbb{R})$	The multiplicative group of invertible 2×2 matrices with real entries
$SL(2, \mathbb{R})$	The multiplicative group of 2×2 matrices with real entries and with determinant 1
S_5	The set of all permutations of $\{1, 2, 3, 4, 5\}$
T	The symmetries of a regular tetrahedron (a good group for chemists!)

This assignment was suggested to me by Professor Annalisa Crannell at Franklin and Marshall, who in turn adapted it from an assignment described by Ralph Czerwinski, "A Writing Assignment in Abstract Algebra", *PRIMUS* **4** no. 2, 117-124 (1994).