Def: An **isometry** of n-dimensional space $\mathbb{R}^n$ is a function from $\mathbb{R}^n$ *onto* $\mathbb{R}^n$ that preserves distance.

Def: Let F be a set of points in $\mathbb{R}^n$. The **symmetry group of** F in $\mathbb{R}^n$ is the set of all isometries of $\mathbb{R}^n$ that carry F onto itself. The group operation is function composition.

December 2, 2002

Questions about Symmetry

1. **What kind of symmetries are there?**

   In $\mathbb{R}^2$, there are four types of isometries:

   (a) Rotation about a point (the *center* of the rotation)

   (b) Reflection across a line (the *axis of reflection*)

   (c) Translation (determined by a *translation vector*)

   (d) Glide Reflection (translation combined with reflection across an axis parallel to the translation vector)

2. **What exactly do we mean by a symmetry anyway?**

   The symmetries of an object F are those **isometries** that map F to itself.

   Recall: An *isometry* of n-dimensional space $\mathbb{R}^n$ is a function from $\mathbb{R}^n$ onto $\mathbb{R}^n$ that preserves distance.

3. **Does the set of symmetries of an object always form a group?**

   Yes!

4. What kinds of groups can be the set of symmetries for some object? Is there some object out there whose set of symmetries is (isomorphic to) $GL(2, \mathbb{R})$? Or A_5?

1. Rotation and translation are orientation-preserving.
2. Reflection and glide-reflection are orientation-reversing, or opposite.

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