Orientation Preserving vs Orientation Reversing

1. Rotation and translation are orientation-preserving.
2. Reflection and glide-reflection are orientation-reversing, or opposite.

1. A composition of orientation-preserving functions is orientation-preserving.
2. A composition of two orientation-reversing functions is orientation-preserving.
3. A composition of one orientation-preserving function and one orientation-reversing function (in either order) is orientation-reversing.

December 4, 2002

Some Unproven But Sensible Facts

1. Rotation about a center followed by rotation about a *different* center is a translation.

2. Reflection across one axis followed by reflection across a *parallel* axis doesn't leave any point in the plane unmoved, and so the composition of two reflections across parallel axes is a translation.

   Reflection across one axis followed by reflection across an intersecting axis leaves only the intersection point unmoved, and so the composition of two reflections across intersecting axes is a rotation.

December 4, 2002

Conclusions About the Order of Different Isometries

1. Rotations have finite order; the order depends on the angle of the rotation. I *believe* the order is $360/gcd(\text{angle}, 360)$.

2. Translations have infinite order.

3. Reflections have order 2.

4. Glide Reflections have infinite order.

December 4, 2002

Conclusions About Finite Plane Symmetry Groups

Let G be a finite plane symmetry group.

1. All elements of G must either be reflections or rotations, because the order of the elements must be finite.

2. All rotations in G must be about the same center point, which we'll call P. This is because the composition of two rotations about different centers would be a translation.

3. Any two reflections in G must have axes that intersect at some point. This is because the composition of two reflections with parallel axes is a translation.

4. All reflections in G have axes that intersect in the same point. This point must also be P, the center point for all rotations in G. This follows from the fact that the composition of two reflections with intersecting axes is a rotation.

Big Conclusion: If G is a finite plane symmetry group, then there exists a point P in the plane that is fixed by every element of G.

Let $S = \{1, 2, 3, 4, 5, 6, 7, 8\}$, and let

$$G = \Big\{ \begin{bmatrix} 1 \end{bmatrix}, \begin{bmatrix} 1 & 3 & 2 \end{bmatrix} \begin{bmatrix} 4 & 6 & 5 \end{bmatrix} \begin{bmatrix} 7 & 8 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 4 & 5 & 6 \end{bmatrix},$$

$$\begin{bmatrix} 1 & 3 & 2 \end{bmatrix} \begin{bmatrix} 4 & 6 & 5 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 3 & \end{bmatrix} \begin{bmatrix} 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 7 & 8 \end{bmatrix}, \begin{bmatrix} 7 & 8 \end{bmatrix} \Big\} \subset S_8.$$

Find

1. $\text{stab}_G(3)$, $\text{orb}_G(3)$
2. $\text{stab}_G(4)$, $\text{orb}_G(4)$
3. $\text{stab}_G(5)$, $\text{orb}_G(5)$
4. $\text{stab}_G(6)$, $\text{orb}_G(6)$
5. $\text{stab}_G(7)$, $\text{orb}_G(7)$
6. $\text{stab}_G(8)$, $\text{orb}_G(8)$

What can we conclude about the orbits of the elements of S?

December 4, 2002