The Orbit Stabilizer Theorem:

Let G be a finite group of permutations of a set S. Then for any i from S, $|G| = |orb_G(i)| \cdot |stab_G(i)|$.

Example:

Let G be the rotation group of a cube in $\mathbb{R}^3$. That is, the group of rotations in $\mathbb{R}^3$ that carry the cube to the cube. Find G. Label the six faces of the cube 1 through 6.

The rotation group of a cube can be regarded as being a group of permutations on the set $\{1, 2, 3, 4, 5, 6\}$. (It may not be all of S_6)

We found $|G|$ using the orbit-stabilizer theorem:

$|orb_G(1)| = 6$, $|stab_G(1)| = 4$, so $|G| = 6 \cdot 4 = 24$.

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