Questions about Symmetry

1. What kind of symmetries are there?

2. **What exactly do we mean by a symmetry anyway?**

   The symmetries of an object F are those **isometries** that map F to itself.

   Recall: An *isometry* of n-dimensional space $\mathbb{R}^n$ is a function from $\mathbb{R}^n$ onto $\mathbb{R}^n$ that preserves distance.

3. **Does the set of symmetries of an object always form a group?**

   Yes!

4. What kinds of groups can be the set of symmetries for some object? Is there some object out there whose set of symmetries is (isomorphic to) $GL(2, \mathbb{R})$? Or A_5?

Example: Reflection across the line connecting $(-1, 2)$ to $(2, 6)$ is an isometry.

Here's what that function looks like (thanks to techniques I learned when I taught linear algebra!):

$$f(x, y) = (-\frac{7x}{25} + \frac{24y}{25} - \frac{16}{5}, \frac{24x}{25} + \frac{7y}{25} + \frac{12}{5}).$$

Def: Let F be a set of points in $\mathbb{R}^n$. The **symmetry group of** F in $\mathbb{R}^n$ is the set of all isometries of $\mathbb{R}^n$ that carry F onto itself. The group operation is function composition.

As we saw Wednesday, this is indeed a group!

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