Let $H \le G$. For any $a \in G$, the set $aH = \{ah | h \in H\}$ is the **left coset of H in G containing** a. Similarly, $Ha = \{ha | h \in H\}$ is the **right coset of H in G containing a.**

Lemma: Let $H \le G$ and $a, b \in G$. Then

1. $a \in aH$. **Proved**
2. $aH = H$ iff $a \in H$. **Proved**
3. Either $aH = bH$ or $aH \cap bH = \emptyset$.
4. $aH = bH$ iff $(a)^{-1}b \in H$.
5. $|aH| = |bH| = |H|$.
6. $aH = Ha$ iff $H = (a)^{-1}Ha$.
7. $aH \le G$ iff $a \in H$.

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1. Property 4 of the lemma says:

$$aH = bH \iff (a)^{-1}b \in H.$$

   Rewrite this property in additive notation. Assume the group is Abelian.

2. Let $H = \{0, \pm 3, \pm 6, \pm 9, \ldots\} = 3\mathbb{Z} \leq \mathbb{Z}$.

   (a) Decide whether or not the following cosets of H in $\mathbb{Z}$ are the same.

      i. $11 + H$ and $17 + H$

      ii. $-1 + H$ and $5 + H$

      iii. $7 + H$ and $23 + H$

   (b) Find all left cosets of H in Z.

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