Let $H \leq G$. For any $a \in G$, the set $aH = \{ah|h \in H\}$ is the **left coset of H in G containing a**. Similarly, $Ha = \{ha|h \in H\}$ is the **right coset of H in G containing a.**

Lemma: Let $H \leq G$ and $a, b \in G$. Then

1. $a \in aH$.
2. $aH = H$ iff $a \in H$.
3. Either $aH = bH$ or $aH \cap bH = \emptyset$.
4. $aH = bH$ iff $(a)^{-1}b \in H$.
5. $|aH| = |bH| = |H|$.
6. $aH = Ha$ iff $H = (a)^{-1}Ha$.
7. $aH \leq G$ iff $a \in H$.

October 28, 2002

1. Property 4 of the lemma says:

$$aH = bH \Longleftrightarrow (a)^{-1}b \in H.$$

   Rewrite this property in additive notation. Assume the group is Abelian.

2. Let $H = \{0, \pm 3, \pm 6, \pm 9, \ldots\} = 3\mathbb{Z} \leq \mathbb{Z}$.

   (a) Decide whether or not the following cosets of H in $\mathbb{Z}$ are the same.

      i. $11 + H$ and $17 + H$

      ii. $-1 + H$ and $5 + H$

      iii. $7 + H$ and $23 + H$

   (b) Find all left cosets of H in Z.

October 28, 2002

Lagrange's Theorem

If G is a finite group and $H \leq G$, then $|H|$ divides $|G|$. Further, the number of distinct left (right) cosets of H in G is $\dfrac{|G|}{|H|}$.

That is, if G is finite, then $|G : H| = \dfrac{|G|}{|H|}$.

October 28, 2002