

# The Fundamental Theorem of Cyclic Groups:

1. Every subgroup of a cyclic group  $\langle a \rangle$  is cyclic.
2. If  $|\langle a \rangle| = n$ , then the order of every subgroup of  $\langle a \rangle$  divides  $n$ .
3. For each divisor  $k$  of  $n$ , there is exactly one subgroup of order  $k$ , namely  $\langle a^{n/k} \rangle$ .

Recall:

## Theorem 4.1

Let  $G$  be a group, and let  $x \in G$ .

1. If  $|x| = \infty$ , then all distinct powers of  $a$  are distinct group elements of  $G$ .
2. If  $|x| = k < \infty$ , then  $\langle x \rangle = \{e, x, x^2, \dots, x^{k-1}\}$ .  
Moreover,

$$x^i = x^j \iff k \text{ divides } i - j.$$

List all the elements of order 8 in  $\mathbb{Z}_{8,000,000}$ . How do you know your list is complete?

September 30, 2002