# The basic structures of Abstract Algebra:

1. **Groups** - sets of elements, with one operation that is associative, has an identity, and has inverses.
   (a) $\mathbb{Z}$ under $+$ is a group
   (b) $\mathbb{Z}_6$ under $+$ is a group
   (c) $\mathbb{Z}$ under $*$ is **not** a group!
2. **Rings** - sets of elements with two operations, at least one of which is as in groups, the other operation need not have inverses.
   (a) $\mathbb{Z}$ under $+$ and $*$
3. **Fields** - rings with additive and multiplicative inverses.
   (a) $\mathbb{Q}$ under $+$ and $*$ is a field
   (b) $\mathbb{R}$ under $+$ and $*$ is a field
   (c) $\mathbb{Z}$ under $+$ and $*$ is **not** a field!

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# Some Examples with elements that are not numbers

1. The set of all symmetries on a square, under composition, is a group.

2. The set of all 2x2 matrices with real entries and non-zero coefficients is a group under matrix multiplication.

3. The set of all polynomials with integer coefficients is a ring (with addition as its first operation and multiplication as its second).

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