Remember the definition of a group: (Here, I'm denoting the operation by addition for later convenience.)

Def: A group G is a non-empty set with a binary operation (denoted by $a+b$) such that for all $a, b, c \in G$,

1. (Remember the hidden requirement: binary means closed under the operation)
2. $a+(b+c) = (a+b)+c$
3. There is an identity $e \in G$ such that $a+e = e+a = a$ for all $a \in G$. (When the operation is addition, we usually denote the identity by 0, so this becomes "there is a $0 \in G$ such that $a+0 = 0+a = a$ for all $a \in G$.)
4. There is an element $-a \in G$ such that $a+(-a) = 0$.

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Prove that the following sets are commutative rings with unity.

1. $R = \{0, 2, 4, 6, 8\}$,under addition and multiplication mod 10.
2. $\mathbb{Z}[\sqrt{2}] = \{a + b\sqrt{2} | a, b \in \mathbb{Z}\}$, under ordinary addition and multiplication of real numbers.

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