Consider $\mathbb{Z}_5 = \{0, 1, 2, 3, 4\}$.

1. Is this a group under addition mod 5?
2. Is this a group under multiplication mod 5?

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What are the elements of $U(12)$?
Write out the Cayley table for $U(12)$.
Is $U(12)$ a group under multiplication mod 12?

Remember: When checking whether a set G is a group, check whether it :

1. *Is closed under the operation:* Let $a, b \in G$. Is $a * b \in G$?

2. *Is associative:* Let $a, b, c \in G$. Does $a * (b * c) = (a * b) * c$?

3. *Has an identity:* Is there an element $e \in G \ni$ for all $a \in G$, $e * a = a = a * e$?

4. *Has inverses:* Let $a \in G$. Is there an element $b \in G$ such that $a * b = e = b * a$?

1. $U(12) = \{1, 5, 7, 11\}$, as these are the only integers greater than or equal to 1 and less than 12 which have no divisors in common with 12 other than 1.

2. Write out the Cayley table for $U(12)$:

Cayley Table for U(12)

$\cdot$ mod 12	1	5	7	11
1	1	5	7	11
5	5	1	11	7
7	7	11	1	5
11	11	7	5	1

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Is $U(12)$ a group under multiplication (mod 12)?

1. **Closed under multiplication mod 12?**

   We can see by looking at the Cayley table that for any $a, b \in U(12), ab \in U(12)$.

2. **Associative?**

   Since multiplication is associative, and since a mod $12 \cdot bmod12 = (ab)mod12$, multiplication mod 12 is also associative.

3. **Identity?**

   Multiplying by 1 mod 12 leaves every number unchanged, and so 1 is the identity.

4. **Inverses?**

   By looking at the Cayley table, I can see that each number has a unique inverse:

   $$(1)^{-1} = 1 \qquad (5)^{-1} = 5 \qquad 7^{-1} = 7 \qquad 11^{-1} = 11$$

   Notice: each number is its own inverse!